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Exact Solutions in Birefringent Fibers for Stochastic N-LSE with Kerr-Law Nonlinearity and Multiplicative White Noise Effects

Khaled. A. Alurrfi [1]*, Ragab M. Almasroub[2], Omar A. Aleyan[3], Marwa A. Saeid[3]

1- Department of Mathematics, Faculty of Science, Elmergib University, Khoms, Libya.

2- Department of Mathematics, Faculty of Science,

Azzyatuna University, Tarhuna, Libya.

3- Department of Mathematics, Faculty of Science,

Alasmarya Islamic University, Zliten, Libya.

E-mail: alurrfi@yahoo.com*, almasroub4392@gmail.com, omar1269@yahoo.com, m.saeid@stu.asmarya.edu.ly

Abstract

This article explores the exact wave solutions of the coupled one-dimensional stochastic non-linear Schrödinger equation with Kerr-law nonlinearity, incorporating multiplicative white noise in the Itô interpretation. The generalized projective Riccati equations (GPREs) scheme is employed to obtain the obtained solutions. The study presents a range of soliton solutions, including dark, bright, singular solitons, in addition to solutions for rational and periodic functions. Finally, the MATLAB software was used to explain the numerical simulation of some soliton solutions.

Keywords: Generalized projective Riccati equations scheme; Non-linear stochastic Schrödinger equation; Exact solutions; Soliton solutions; White noise.

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الحلول الدقيقة في الألياف ثنائية الانكسار لمعادلة شرودنجر غير الخطية العشوائية مع قانون كير اللاخطي وتأثيرات الضوضاء البيضاء المضاعفة

خالد علي العرفي [1]*، رجب محمد المسروب [2]، عمر علي العيان[3]، مروة عبد السلام سعيد[3]

[1]قسم الرياضيات، كلية العلوم، جامعة المرقب، الخمس، ليبيا.

[2]قسم الرياضيات، كلية العلوم، جامعة الزيتونة، ترهونة، ليبيا.

[3]قسم الرياضيات، كلية العلوم، الجامعة الأسمرية الإسلامية، زليتن، ليبيا.

* Corresponding author: alurrfi@yahoo.com

الملخص

يستكشف هذا المقال الحلول الموجية الدقيقة لمعادلة شرودنجر غير الخطية العشوائية أحادية البعد المقترنة مع عدم الخطية وفقًا لقانون كير، مع دمج الضوضاء البيضاء المضاعفة في تفسير إيتو. يتم استخدام مخطط معادلات ريكاتي الإسقاطية المعممة (GPREs)للحصول على الحلول التي تم الحصول عليها. تقدم الدراسة مجموعة من حلول سوليتونية، بما في ذلك سوليتونات dark وbright وsingular، بالإضافة إلى حلول الدوال النسبية والدورية. أخيرًا، تم استخدام برنامج MATLAB لشرح المحاكاة العددية لبعض حلول سولوتونية.

الكلمات المفتاحية: طريقة معادلات ريكاتي الإسقاطية المعممة؛ معادلة شرودنجر العشوائية غير الخطية؛ الحلول الدقيقة؛ حلول سوليتونية؛ الضوضاء البيضاء.

1. Introduction

Non-linear systems of partial differential equations (PDEs) are useful for explaining a variety of phenomena in a variety of physical scientific domains, including economy, biology, fluid mechanics, non-linear optical fibers, and plasma physics. Lately, many researchers have focused on finding explicit soliton solutions for non-linear PDEs using different approaches. Recently, a novel idea known as stochasticity has emerged. One of the characteristics of the stochasticity component is the existence of several papers that

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examine this idea [1–12]. Optics, quantum mechanics, deep water, biophysics, plasma physics, electro-magnetic wave propagation, and many other non-linear fields use the non-linear Schrödinger equation (N-LSE), which offers solitary type solutions, as the primary representative method for characterizing wave behaviors. Various successful approaches, including the (G'/G) -expansion method [13-16], the tanh-function expansion method [17, 18], the new mapping method [19, 20], the enhanced Kudryashov method [21, 22], the generalized projective Riccati equations scheme [23-25], the $(G^{'}/G, 1/G)$ -expansion method [26, 27] and Lie symmetry method [28, 29] have been utilized to generate soliton solutions and other exact wave solutions for the NLPDEs.

In polarization preserving fibers, the coupled one-dimensional N-LSE with spatio-temporal dispersion (STD) and zero noise has been discussed in [30, 31]. In polarization preserving fibers, the coupled one-dimensional N-LSE with Kerr-law nonlinearity by multiplicative noise in Itô sense has been discussed in [7] using the new extended auxiliary equation method.

The aim of this article is to study the coupled one-dimensional stochastic N-LSE with Kerr-law nonlinearity in birefringent fibers with multiplicative white noise instead, which would yield soliton solutions with the effect of Wiener process included. The approach utilized to obtain solitons and other exact solutions is the generalized projective Riccati equations (GPREs) scheme.

This article is structured as follows: In section 2, the governing model is introduced. In section 3, wave transformation and the mathematical analysis of equations (2) and (3) is presented. In section 4, the description of the generalized projective Riccati equations scheme is given. In section 5, we will solve the coupled one-dimensional N-LSE in birefringent fibers (2) and (3) using the generalized projective Riccati equations scheme. Finally, some conclusions are illustrated in section 6.

2. The Governing Stochastic Non-linear Schrödinger Equation

According to the Itô interpretation, the single form of stochastic one-dimensional N-LSE in polarization-preserving fibers with multiplicative white noise and Kerr-law nonlinearity is expressed as [7]:

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$$iH_t(x,t) + \alpha H_{xx}(x,t) + \beta|H(x,t)|^2 H(x,t) + \sigma \frac{dW(t)}{dt} H(x,t) = 0,\ i = \sqrt{-1}, \quad (1)$$

In this context, $H(x,t)$ denote a complex-valued function associated with the wave profile. The variables $t,\ x,\ \alpha,\ \beta$ and σ correspond to the temporal variable, spatial variable, coefficients of the group velocity dispersion (GVD) term, coefficients of the non-linear dispersion term, and noise strength, respectively. It is worth noting that $\alpha,\ \beta$ and σ are real values. While, $W(t)$ denotes the one-dimensional standard Wiener process, which is defined by the integral known as the Itô integral:

$$W'(t) = \frac{dW(t)}{dt}, W(t) = \int_0^t H(\tau) dW(\tau),\ \tau < t,$$

where τ is the stochastic variable .
Within the birefringent fibers, equation (1) undergoes splitting into two different components for the first appearance:

$$iu_t + \alpha_1 u_{xx} + (\beta_1|u|^2 + \gamma_1|v|^2)u + \sigma \frac{dW(t)}{dt} u = 0, \quad (2)$$

and

$$iv_t + \alpha_2 v_{xx} + (\beta_2|v|^2 + \gamma_2|u|^2)v + \sigma \frac{dW(t)}{dt} v = 0, \quad (3)$$

In wave dynamics, the profiles of two different waves are described by the variables $u(x,t)$ and $v(x,t)$ denote complex-valued functions.
Here, x and t represents the spatial and temporal variables respectively. The constants α_j $(j = 1,2)$ are the coefficients of GVD. β_j $(i = 1,2)$ are the coefficients of self-phase modulation (SPM) and γ_j $(j = 1,2)$ represent the cross-phase modulation (XPM) terms respectively. Finally, σ represents white noise coefficient.

3. Wave transformation and the mathematical analysis

For converting equations (2) and (3) to ordinary differential equations (ODEs), we presume that the wave solutions have the forms:

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$$u(x,t) = \varphi_1(\eta)\exp[i(-ax + bt + c_0 + \sigma W(t))], \tag{4}$$

$$v(x,t) = \varphi_2(\eta)\exp[i(-ax + bt + c_0 + \sigma W(t))], \tag{5}$$

$$\eta = x - \omega t, \tag{6}$$

where a, b and c_0 are nonzero real-valued constants. From the phase component, a gives the frequency of the solitons, while b is the wave number and ω is the velocity soliton. The functions $\varphi_j(\eta)$ for $j = 1,2$ is real functions which represent the amplitude portions of the solitons and the phase components of the solitons, respectively. Inserting (4) and (5) into equations (2) and (3) gives the real parts:

$$\alpha_1\varphi_1'' - (a^2\alpha_1 + b)\varphi_1 + \gamma_1\varphi_1\varphi_2^2 + \beta_1\varphi_1^3 = 0, \tag{7}$$

and

$$\alpha_2\varphi_2'' - (a^2\alpha_2 + b)\varphi_2 + \gamma_2\varphi_2\varphi_1^2 + \beta_2\varphi_2^3 = 0, \tag{8}$$

while the imaginary parts are:

$$(2a\alpha_1 + \omega)\varphi_1' = 0, \tag{9}$$

and

$$(2a\alpha_2 + \omega)\varphi_2' = 0. \tag{10}$$

By applying the principle of linearly independence on (9) and (10) to get the wave number ω:

$$\omega = -2a\alpha_j,\ j = 1,2. \tag{11}$$

For simplicity's sake, let's now put

$$\varphi_2(\eta) = \Theta\varphi_1(\eta), \tag{12}$$

where Θ is a non-zero constant, such that $\Theta \neq 1$. Equations (7) and (8) can be reduced as:

$$\alpha_1\varphi_1'' - (a^2\alpha_1 + b)\varphi_1 + (\chi^2\gamma_1 + \beta_1)\varphi_1^3 = 0, \tag{13}$$

and

$$\alpha_2\varphi_1'' - (a^2\alpha_2 + b)\varphi_1 + (\gamma_2 + \chi^2\beta_2)\varphi_1^3 = 0. \tag{14}$$

Equations (13) and (14) are equivalent under the constraint conditions:

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$$\alpha_1 = \alpha_2, \tag{15}$$

$$a^2\alpha_1 + b = a^2\alpha_2 + b, \quad \chi^2\gamma_1 + \beta_1 = \gamma_2 + \chi^2\beta_2. \tag{16}$$

In the next sections, we will construct the solitons and other exact wave solutions of equations (2) and (3) by using the generalized projective Riccati equations scheme.

4. Outline of the generalized projective Riccati equations (GPREs) scheme

Consider of the NLPDE in the form:

$$\aleph(u, u_x, u_t, u_{xx}, u_{tt}, \dots) = 0, \tag{17}$$

Where $u = u(x, t)$ is an unknown function, $\aleph$ is a polynomial in $u(x, t)$ and its partial derivatives in which the highest order derivatives and non-linear terms are involved. Here, we outline the fundamental steps of the GPREs scheme [24, 25]:

Step 1. By applying the following transformation:

$$u(x, t) = u(\eta), \quad \eta = x - \omega t, \tag{18}$$

To reduce equation (17) to the following non-linear ODE:

$$\Im(u, u', u'', \dots) = 0, \tag{19}$$

Where ω is velocity of the propagation, $\Im$ is a polynomial of $u(\eta)$ and its total derivatives $u'(\eta),\ u''(\eta), \dots$ and $' = \frac{d}{d\eta}$.

Step 2. We presume that the solution of equation (19) has the form

$$u(\eta) = A_0 + \sum_{i=1}^{N} \sigma^{i-1}(\eta)[A_i\sigma(\eta) + B_i\tau(\eta)], \tag{20}$$

Where A_0, A_i and B_i are constants to be determined with the condition that $A_N^2 + B_N^2 \neq 0$. The functions, $\sigma(\eta)$ and $\tau(\eta)$ satisfy the ODEs:

$$\sigma'(\eta) = \varepsilon\sigma(\eta)\tau(\eta), \tag{21}$$

$$\tau'(\eta) = \Upsilon + \varepsilon\tau^2(\eta) - \mu\sigma(\eta), \quad \varepsilon = \pm 1, \tag{22}$$

Where

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$$\tau^2(\eta) = -\varepsilon\left(\Upsilon - 2\mu\sigma(\eta) + \frac{\mu^2 + r}{\Upsilon}\sigma^2(\eta)\right), \tag{23}$$

Here, $r = \pm 1$ and Υ, μ are constants different from zero.
If $\Upsilon = \mu = 0$, equation (19) has the formal solution:

$$u(\eta) = \sum_{i=1}^{N} A_i \tau^i(\eta), \tag{24}$$

Where $\tau(\eta)$ satisfies the non-linear ODE:

$$\tau'(\eta) = \tau^2(\eta). \tag{25}$$

Step 3. The positive integer number N in (20) must be determine by using the homogeneous balance between the highest-order derivatives and the highest-non-linear terms in equation (19).

Step 4. Substitute (20) along with equations (21)-(23) into equation (19). Collecting all terms of the same order of $\sigma^j(\eta)\tau^i(\eta)$ $(j = 0,1, \ldots; i = 0,1)$. Setting each coefficient to zero, yields a set of algebraic equations which can be solved to find the values of A_0, A_i, B_i, ω, μ and Υ.

Step 5. It is well known that equations (21) and (22) admits the following solutions [24, 25]:

Case 1. If $\varepsilon = -1$, $r = -1$, $\Upsilon > 0$,

$$\sigma_1(\eta) = \frac{\Upsilon\,\mathrm{sech}(\sqrt{\Upsilon}\eta)}{\mu\,\mathrm{sech}(\sqrt{\Upsilon}\eta) + 1}, \tau_1(\eta) = \frac{\sqrt{\Upsilon}\tanh(\sqrt{\Upsilon}\eta)}{\mu\,\mathrm{sech}(\sqrt{\Upsilon}\eta) + 1}, \tag{26}$$

Case 2. If $\varepsilon = -1$, $r = 1$, $\Upsilon > 0$,

$$\sigma_2(\eta) = \frac{\Upsilon\,\mathrm{csch}(\sqrt{\Upsilon}\eta)}{\mu\,\mathrm{csch}(\sqrt{\Upsilon}\eta) + 1}, \tau_2(\eta) = \frac{\sqrt{\Upsilon}\coth(\sqrt{\Upsilon}\eta)}{\mu\,\mathrm{csch}(\sqrt{\Upsilon}\eta) + 1}. \tag{27}$$

Case 3. If $\varepsilon = 1$, $r = -1$, $\Upsilon > 0$,

$$\sigma_3(\eta) = \frac{\Upsilon\sec(\sqrt{\Upsilon}\eta)}{\mu\sec(\sqrt{\Upsilon}\eta) + 1}, \tau_3(\eta) = \frac{\sqrt{\Upsilon}\tan(\sqrt{\Upsilon}\eta)}{\mu\sec(\sqrt{\Upsilon}\eta) + 1}, \tag{28}$$

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$$\sigma_4(\eta) = \frac{\Upsilon \csc(\sqrt{\Upsilon}\eta)}{\mu \csc(\sqrt{\Upsilon}\eta) + 1}, \tau_4(\eta) = -\frac{\sqrt{\Upsilon} \cot(\sqrt{\Upsilon}\eta)}{\mu \csc(\sqrt{\Upsilon}\eta) + 1}. \quad (29)$$

Case 4. If $\Upsilon = \mu = 0$,

$$\sigma_5(\eta) = \frac{C}{\eta}, \tau_5(\eta) = \frac{1}{\varepsilon\eta}, \quad (30)$$

Where C is constant different from zero.

Step 6. Substituting the values of A_0, A_i, B_i, ω, μ and Υ as well as the solutions (26)-(30) into (20) we reach the exact solutions of equation (17).

5. Solitons and various exact wave solutions to equations (1) and (2) using the generalized projective Riccati equations (GPREs) scheme

In this section, we apply the so called generalized projective Riccati equations scheme described in Sec. 4 to find many new soliton, periodic and rational solutions of equations (1) and (2).

Balancing φ_1'' with φ_1^3 in equation (13), then the following one attains:

$$N + 2 = 3N \Rightarrow N = 1. \quad (31)$$

From (20), the formal solution of equation (13) has the following form:

$$\varphi(\eta) = A_0 + A_1\sigma(\eta) + B_1\tau(\eta), \quad (32)$$

Where A_0, A_1 and B_1 are real constants to be determined with the condition that $A_1 \neq 0$ or $B_1 \neq 0$, while the functions, $\sigma(\eta)$ and $\tau(\eta)$ satisfy the ODEs (21) and (22).

By inserting (32) into equation (13) and then aggregating all the coefficients of $\sigma^j(\eta)\tau^i(\eta)$ ($j = 0,1,2,3$ and $i = 0,1,2,3$). Setting the coefficients of equal to zero, yields the following system of algebraic equations:

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$$\sigma^3 \ : \ A_1^3\chi^2\gamma_1 + A_1^3\beta_1 - \frac{3\varepsilon A_1 B_1^2\beta_1\mu^2}{\Upsilon} - \frac{3\varepsilon A_1 B_1^2\beta_1 r}{\Upsilon} - \frac{2\varepsilon^3 A_1\alpha_1\mu^2}{\Upsilon} - \frac{2\varepsilon^3 A_1\alpha_1 r}{\Upsilon} - \frac{3\varepsilon\chi^2 A_1 B_1^2\gamma_1\mu^2}{\Upsilon} - \frac{3\varepsilon\chi^2 A_1 B_1^2\gamma_1 r}{\Upsilon} = 0,$$

$$\sigma^2 \ : \ 3A_0A_1^2\chi^2\gamma_1 + 4\varepsilon^3 A_1\alpha_1\mu - \alpha_1 A_1\varepsilon\mu + 3A_0A_1^2\beta_1 - \frac{3\varepsilon A_0 B_1^2\beta_1\mu^2}{\Upsilon} - \frac{3\varepsilon A_0 B_1^2\beta_1 r}{\Upsilon} + 6\varepsilon\chi^2 A_1 B_1^2\gamma_1\mu + 6\varepsilon A_1 B_1^2\beta_1\mu - \frac{3\varepsilon\chi^2 A_0 B_1^2\gamma_1\mu^2}{\Upsilon} - \frac{3\varepsilon\chi^2 A_0 B_1^2\gamma_1 r}{\Upsilon} = 0,$$

$$\sigma^2\tau \ : \ 3A_1^2B_1\beta_1 - \frac{\varepsilon\mu^2 B_1^3\beta_1}{\Upsilon} - \frac{\varepsilon r B_1^3\beta_1}{\Upsilon} - \frac{2\varepsilon^3\mu^2 B_1\alpha_1}{\Upsilon} - \frac{2\varepsilon^3 r B_1\alpha_1}{\Upsilon} + 3\chi^2A_1^2B_1\gamma_1 - \frac{\varepsilon r\chi^2 B_1^3\gamma_1}{\Upsilon} - \frac{\varepsilon\mu^2\chi^2 B_1^3\gamma_1}{\Upsilon} = 0,$$

$$\sigma \ : -2\Upsilon\varepsilon^3 A_1\alpha_1 + 3\chi^2A_0^2A_1\gamma_1 + \Upsilon\varepsilon A_1\alpha_1 - a^2A_1\alpha_1 + 3A_0^2A_1\beta_1 - bA_1 + 6\chi^2\mu\varepsilon A_0B_1^2\gamma_1 - 3\Upsilon\chi^2\varepsilon A_1B_1^2\gamma_1 + 6\mu\varepsilon A_0B_1^2\beta_1 - 3\Upsilon\varepsilon A_1B_1^2\beta_1 = 0,$$

$$\sigma\tau \ : \ 2\chi^2\mu\varepsilon B_1^3\gamma_1 + 6\chi^2A_0A_1B_1\gamma_1 + 4\mu\varepsilon^3B_1\alpha_1 + 2\mu\varepsilon B_1^3\beta_1 - 3\mu\varepsilon B_1\alpha_1 + 6A_0A_1B_1\beta_1 = 0,$$

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$$\begin{aligned}\tau\ :&-\Upsilon\chi^2\varepsilon B_1^3\gamma_1-2\Upsilon\varepsilon^3B_1\alpha_1-\Upsilon\varepsilon B_1^3\beta_1+3\chi^2A_0^2B_1\gamma_1\\&+2\Upsilon\varepsilon B_1\alpha_1-a^2B_1\alpha_1+3A_0^2B_1\beta_1-bB_1=0,\\\sigma^0:&-3\varepsilon A_0B_1^2\beta_1\Upsilon+A_0^3\beta_1-A_0b+A_0^3\chi^2\gamma_1-A_0a^2\alpha_1\\&-3\varepsilon\chi^2A_0B_1^2\gamma_1\Upsilon=0.\end{aligned}\tag{33}$$

According to Step 5, of Section 4, there are three cases of solutions of the algebraic equations (33) to be discussed as follows:

Case 1. Putting $\varepsilon=-1,\ r=-1$ in the above algebraic equations (33) and solving them by Maple, we have the following results:

Result 1.

$$\begin{gathered}\Upsilon=\frac{a^2\alpha_1+b}{\alpha_1},\ \mu=0,\ A_0=0,\\A_1=\sqrt{\frac{2\alpha_1^2}{(\chi^2\gamma_1+\beta_1)(a^2\alpha_1+b)}},\ B_1=0,\end{gathered}\tag{34}$$

where $\alpha_1(a^2\alpha_1+b)>0$ and $(a^2\alpha_1+b)(\chi^2\gamma_1+\beta_1)>0$. Superseding Eq. (34) into Eq. (32), we reach the bright-soliton (see Figure 1.):

$$u(x,t)=\sqrt{\frac{2(a^2\alpha_1+b)}{(\chi^2\gamma_1+\beta_1)}}\left[\operatorname{sech}\left(\sqrt{\frac{a^2\alpha_1+b}{\alpha_1}}\eta\right)\right]e^{i[-ax+bt+c_0+\sigma W(t)]},\tag{35}$$

$$v(x,t)=\Theta\left\{\sqrt{\frac{2(a^2\alpha_1+b)}{(\chi^2\gamma_1+\beta_1)}}\left[\operatorname{sech}\left(\sqrt{\frac{a^2\alpha_1+b}{\alpha_1}}\eta\right)\right]\right\}e^{i[-ax+bt+c_0+\sigma W(t)}\tag{36}$$

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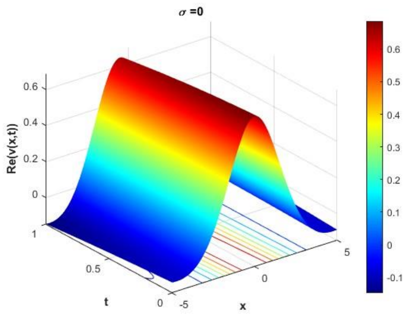
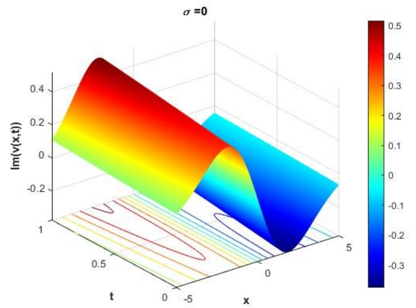
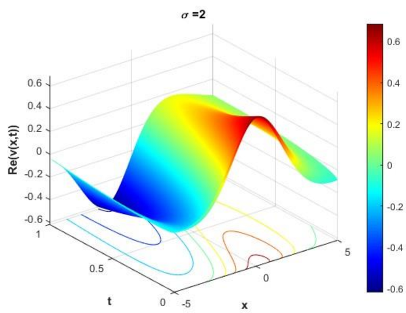


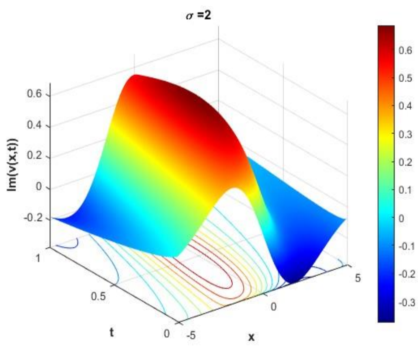
$Re(v(x,t))$
(a)



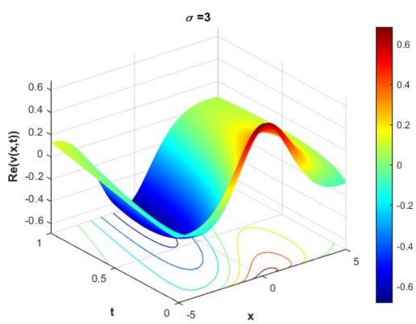
Im(v(x,t))
(b)



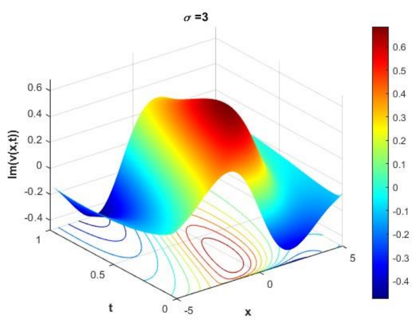
$Re(v(x,t))$
(c)



$Im(v(x,t))$
(d)



$Re(v(x,t))$
(e)



$Im(v(x,t))$
(f)

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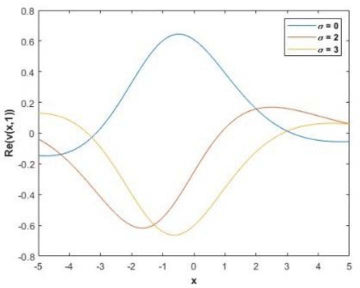
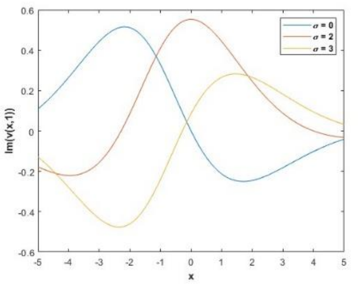


Re(v(x,1))
(g)



$Im(v(x,1))$
(h)

Fig . 1. Representation of the bright-soliton solutions (36) in 2D and 3D graphs . These figures are obtained by $a = \chi = 0.5, b = 0$, $\alpha = \gamma = 1$, $\beta = 4, c_0 = 0, \Theta = 2$ and $W(t) = \sqrt{t}$. Additionally, various values of the noise effect strength coefficient σ are appear in graphs (a) − (h).

Result 2.

$$\Upsilon = -\frac{a^2\alpha_1 + b}{2\alpha_1}, \ \mu = 0, \ A_0 = 0, \ A_1 = 0, \qquad (37)$$

$$B_1 = \sqrt{-\frac{2\alpha_1}{\chi^2\gamma_1 + \beta_1}},$$

where $\alpha_1(a^2\alpha_1 + b) < 0$ and $\alpha_1(\chi^2\gamma_1 + \beta_1) < 0$.
In this result, we deduce the dark-soliton (see Figure 2.) :

$$u(x,t) = \sqrt{\frac{a^2\alpha_1 + b}{\chi^2\gamma_1 + \beta_1}}\left[\tanh\left(\sqrt{-\frac{a^2\alpha_1 + b}{2\alpha_1}}\eta\right)\right]e^{i[-ax+bt+c_0+\sigma W(t)]}, \qquad (38)$$

$$v(x,t) = \Theta\left\{\sqrt{\frac{a^2\alpha_1 + b}{\chi^2\gamma_1 + \beta_1}}\left[\tanh\left(\sqrt{-\frac{a^2\alpha_1 + b}{2\alpha_1}}\eta\right)\right]\right\}e^{i[-ax+bt+c_0+\sigma W(t)]}, \qquad (39)$$

provided $(a^2\alpha_1 + b)(\chi^2\gamma_1 + \beta_1) > 0$.

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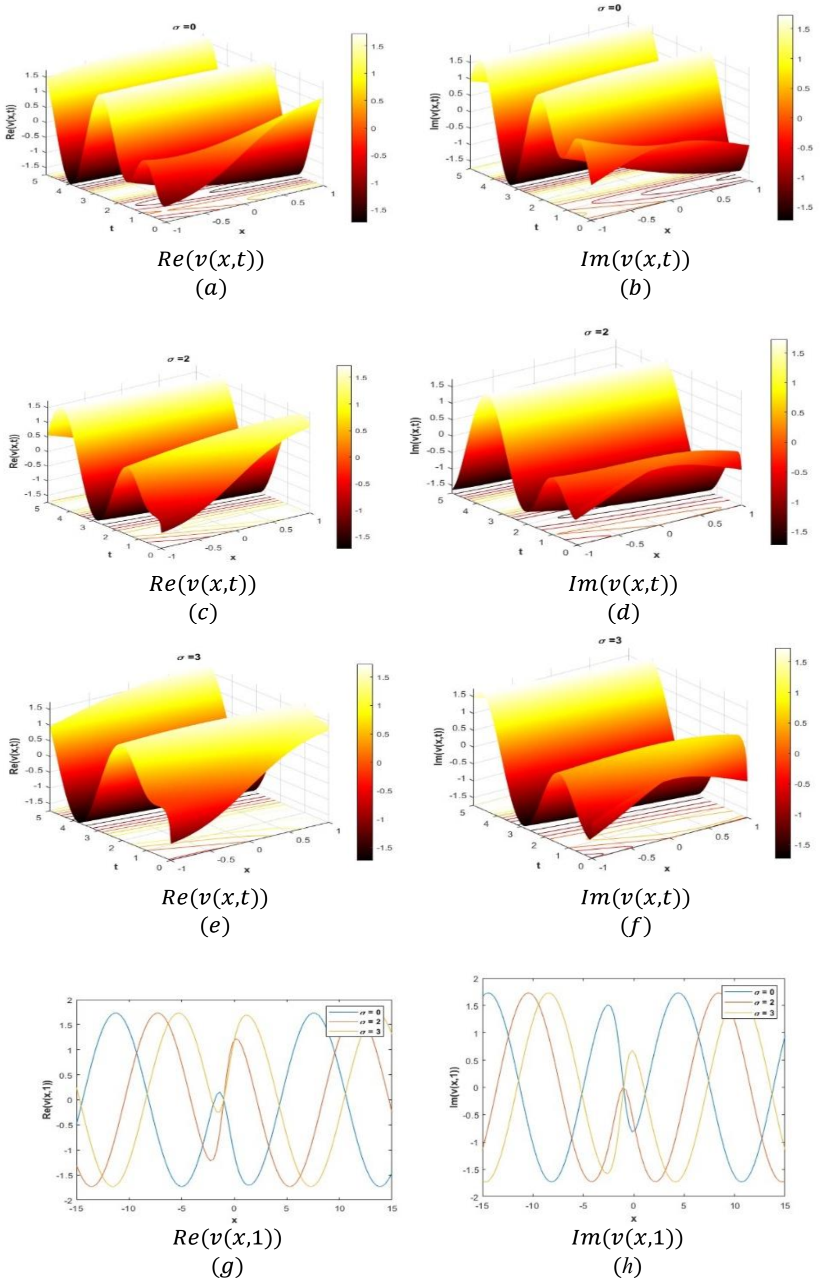

$Re(v(x,t))$ (a) — $Im(v(x,t))$ (b)

$Re(v(x,t))$ (c) — $Im(v(x,t))$ (d)

$Re(v(x,t))$ (e) — $Im(v(x,t))$ (f)

$Re(v(x,1))$ (g) — $Im(v(x,1))$ (h)

Fig. 2. Representation of the dark-soliton solutions (39) in 2D and 3D graphs . These figures are obtained by $a = 0.5, b = -2.5,\ \alpha = \gamma = \chi = 1, \beta = -4, c_0 = 0, \Theta = 2$ and $W(t) = \sqrt{t}$. Additionally, various values of the noise effect strength coefficient σ are appear in graphs $(a) - (h)$.

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Result 3.

$$\Upsilon = -\frac{2(a^2\alpha_1 + b)}{\alpha_1},\ \mu = \mu,\ A_0 = 0,$$
$$A_1 = \sqrt{\frac{\alpha_1^2(\mu^2-1)}{4(\chi^2\gamma_1+\beta_1)(a^2\alpha_1+b)}}, B_1 = \sqrt{-\frac{\alpha_1}{2(\chi^2\gamma_1+\beta_1)}}, \tag{40}$$

where $\alpha_1(a^2\alpha_1 + b) < 0,\ (\mu^2 - 1)(\chi^2\gamma_1 + \beta_1)(a^2\alpha_1 + b) > 0$ and $\alpha_1(\chi^2\gamma_1 + \beta_1) < 0$.
This result arrives the combo dark- bright soliton (see Figure 3.):

$$u(x,t) = \left\{-\sqrt{\frac{(\mu^2-1)(a^2\alpha_1+b)}{\chi^2\gamma_1+\beta_1}}\left(\frac{\operatorname{sech}\left(\sqrt{-\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)}{\mu\operatorname{sech}\left(\sqrt{-\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)+1}\right) + \sqrt{\frac{a^2\alpha_1+b}{\chi^2\gamma_1+\beta_1}}\left(\frac{\tanh\left(\sqrt{-\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)}{\mu\operatorname{sech}\left(\sqrt{-\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)+1}\right)\right\} e^{i[-ax+bt+c_0+\sigma W(t)]}, \tag{41}$$

$$v(x,t) = \Theta\left\{-\sqrt{\frac{(\mu^2-1)(a^2\alpha_1+b)}{\chi^2\gamma_1+\beta_1}}\left(\frac{\operatorname{sech}\left(\sqrt{-\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)}{\mu\operatorname{sech}\left(\sqrt{-\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)+1}\right) + \sqrt{\frac{a^2\alpha_1+b}{\chi^2\gamma_1+\beta_1}}\left(\frac{\tanh\left(\sqrt{-\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)}{\mu\operatorname{sech}\left(\sqrt{-\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)+1}\right)\right\} e^{i[-ax+bt+c_0+\sigma W(t)]}, \tag{42}$$

provided $(\mu^2 - 1)(a^2\alpha_1 + b)(\chi^2\gamma_1 + \beta_1) > 0$.

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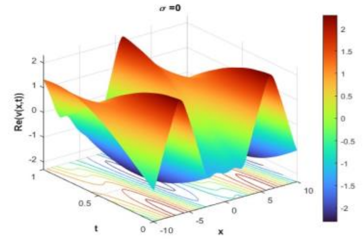
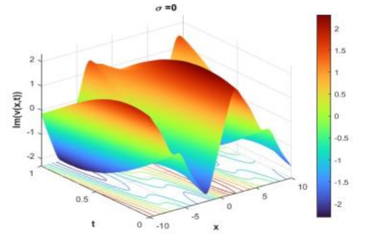
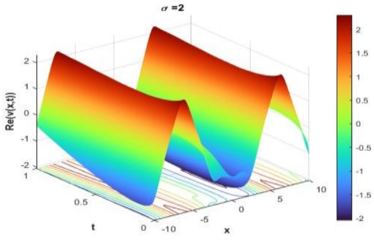
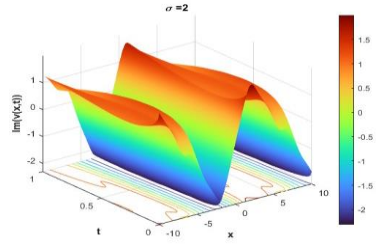




$Re(v(x,t))$
(a)

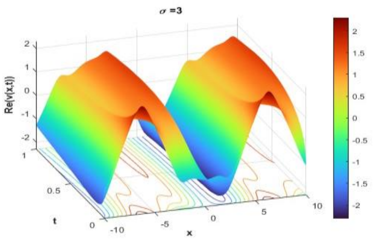
$Im(v(x,t))$
(b)

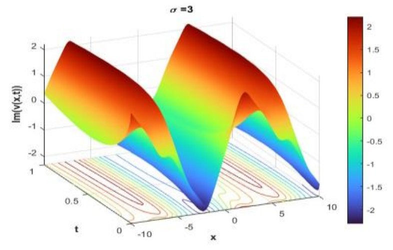




$Re(v(x,t))$
(c)

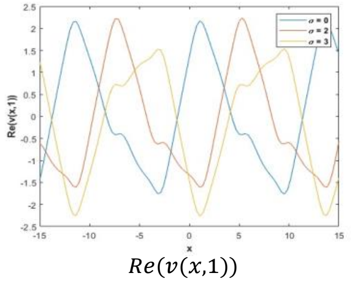
$Im(v(x,t))$
(d)

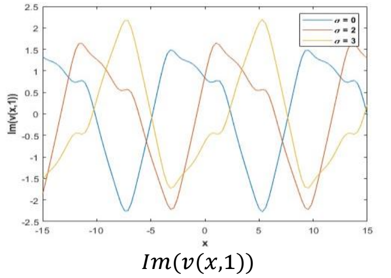




$Re(v(x,t))$
(e)

$Im(v(x,t))$
(f)





$Re(v(x,1))$
(g)

$Im(v(x,1))$
(h)

Fig. 3. Representation of dark- bright soliton solutions (42) in 2D and 3D graphs. These figures are obtained by $a = 0.5, b = -2.25, \mu = 3$, $\alpha = \gamma = \chi = 1$, $\beta = -4, c_0 = 0, \Theta = 2$ and $W(t) = \sqrt{t}$. Additionally, various values of the noise effect strength coefficient σ are appear in graphs $(a) - (h)$.

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Case 2. Putting $\varepsilon = -1, \ r = 1$ in the algebraic equations (33) and solving them by Maple, we get the following results:

Result 1.

$$\Upsilon = \frac{a^2\alpha_1 + b}{\alpha_1}, \ \mu = 0, \ A_0 = 0,$$
$$A_1 = \sqrt{-\frac{2\alpha_1^2}{(\chi^2\gamma_1 + \beta_1)(a^2\alpha_1 + b)}}, \ B_1 = 0, \tag{43}$$

where $\alpha_1(a^2\alpha_1 + b) > 0$ and $(\chi^2\gamma_1 + \beta_1)(a^2\alpha_1 + b) < 0$.
From (43), we construct the singular soliton:

$$u(x,t) = \sqrt{-\frac{2(a^2\alpha_1 + b)}{(\chi^2\gamma_1 + \beta_1)}}\left[\operatorname{csch}\left(\sqrt{\frac{a^2\alpha_1 + b}{\alpha_1}}\eta\right)\right]e^{i[-ax+bt+c_0+\sigma W(t)]}, \tag{44}$$

$$v(x,t) = \Theta\left\{\sqrt{-\frac{2(a^2\alpha_1 + b)}{(\chi^2\gamma_1 + \beta_1)}}\left[\operatorname{csch}\left(\sqrt{\frac{a^2\alpha_1 + b}{\alpha_1}}\eta\right)\right]\right\}e^{i[-ax+bt+c_0+\sigma W(t)]}. \tag{45}$$

Result 2.

$$\Upsilon = -\frac{a^2\alpha_1 + b}{2\alpha_1}, \ \mu = 0, \ A_0 = 0, \ A_1 = 0,$$
$$B_1 = \sqrt{-\frac{2\alpha_1}{\chi^2\gamma_1 + \beta_1}}, \tag{46}$$

where $\alpha_1(a^2\alpha_1 + b) < 0$ and $\alpha_1(\chi^2\gamma_1 + \beta_1) < 0$.
In this result, we deduce the singular soliton:

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$$u(x,t)=\sqrt{\frac{a^2\alpha_1+b}{\chi^2\gamma_1+\beta_1}}\left[\coth\left(\sqrt{-\frac{a^2\alpha_1+b}{2\alpha_1}}\eta\right)\right]e^{i[-ax+bt+c_0+\sigma W(t)]}, \quad (47)$$

$$v(x,t)=\Theta\left\{\sqrt{\frac{a^2\alpha_1+b}{\chi^2\gamma_1+\beta_1}}\left[\coth\left(\sqrt{-\frac{a^2\alpha_1+b}{2\alpha_1}}\eta\right)\right]\right\}e^{i[-ax+bt+c_0+\sigma W(t)]}, \quad (48)$$

provided $(a^2\alpha_1+b)(\chi^2\gamma_1+\beta_1)>0.$

Result 3.

$$\Upsilon=-\frac{2(a^2\alpha_1+b)}{\alpha_1},\ \mu=\mu,\ A_0=0,$$
$$A_1=\sqrt{\frac{\alpha_1^2(\mu^2+1)}{4(\chi^2\gamma_1+\beta_1)(a^2\alpha_1+b)}},\ B_1=\sqrt{-\frac{\alpha_1}{2(\chi^2\gamma_1+\beta_1)}}, \quad (49)$$

where $\alpha_1(a^2\alpha_1+b)<0$, $(a^2\alpha_1+b)(\chi^2\gamma_1+\beta_1)>0$ and $\alpha_1(\chi^2\gamma_1+\beta_1)<0.$
This result arrives the combo-singular soliton:

$$u(x,t)=\left\{-\sqrt{\frac{(\mu^2+1)(a^2\alpha_1+b)}{(\chi^2\gamma_1+\beta_1)}}\left(\frac{\operatorname{csch}\left(\sqrt{-\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)}{\mu\operatorname{csch}\left(\sqrt{-\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)+1}\right)+\sqrt{\frac{a^2\alpha_1+b}{\chi^2\gamma_1+\beta_1}}\left(\frac{\coth\left(\sqrt{-\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)}{\mu\operatorname{csch}\left(\sqrt{-\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)+1}\right)\right\}e^{i[-ax+bt+c_0+\sigma W(t)]}, \quad (50)$$

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$$v(x,t)=\Theta\left\{-\sqrt{\frac{(\mu^2+1)(a^2\alpha_1+b)}{(\chi^2\gamma_1+\beta_1)}}\left(\frac{\operatorname{csch}\left(\sqrt{-\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)}{\mu\operatorname{csch}\left(\sqrt{-\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)+1}\right)+\sqrt{\frac{a^2\alpha_1+b}{\chi^2\gamma_1+\beta_1}}\left(\frac{\coth\left(\sqrt{-\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)}{\mu\operatorname{csch}\left(\sqrt{-\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)+1}\right)\right\}e^{i[-ax+bt+c_0+\sigma W(t)]}, \quad (51)$$

provided $(a^2\alpha_1+b)(\chi^2\gamma_1+\beta_1)>0$.

Case 3. Putting $\varepsilon=1,\ \ r=-1$ in the algebraic equations (33) and solving them by Maple, we have the following results:

Result 1.

$$\Upsilon=-\frac{a^2\alpha_1+b}{\alpha_1},\ \mu=0,\ A_0=0,$$
$$A_1=\sqrt{\frac{2\alpha_1^2}{(\chi^2\gamma_1+\beta_1)(a^2\alpha_1+b)}},\ B_1=0, \quad (52)$$

where $\alpha_1(a^2\alpha_1+b)<0$ and $(a^2\alpha_1+b)(\chi^2\gamma_1+\beta_1)>0$. From (52), we obtain the periodic:

$$u(x,t)=-\sqrt{\frac{2(a^2\alpha+b)}{\chi^2\gamma_1+\beta_1}}\sec\left(\sqrt{-\frac{a^2\alpha_1+b}{\alpha_1}}\eta\right)e^{i[-ax+bt+c_0+\sigma W(t)]}, \quad (53)$$

$$v(x,t)=\Theta\left\{-\sqrt{\frac{2(a^2\alpha+b)}{\chi^2\gamma_1+\beta_1}}\sec\left(\sqrt{-\frac{a^2\alpha_1+b}{\alpha_1}}\eta\right)\right\}e^{i[-ax+bt+c_0+\sigma W(t)]}, \quad (54)$$

and

$$u(x,t)=-\sqrt{\frac{2(a^2\alpha+b)}{\chi^2\gamma_1+\beta_1}}\csc\left(\sqrt{-\frac{a^2\alpha_1+b}{\alpha_1}}\eta\right)e^{i[-ax+bt+c_0+\sigma W(t)]}, \quad (55)$$

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$$v(x,t) = \Theta\left\{-\sqrt{\frac{2(a^2\alpha + b)}{\chi^2\gamma_1 + \beta_1}}\csc\left(\sqrt{-\frac{a^2\alpha_1 + b}{\alpha_1}}\eta\right)\right\}e^{i[-ax+bt+c_0+\sigma W(t)]}. \quad (56)$$

Result 2.

$$\Upsilon = \frac{a^2\alpha_1 + b}{2\alpha_1},\ \mu = 0,\ A_0 = 0,\ A_1 = 0, \quad (57)$$
$$B_1 = \sqrt{-\frac{2\alpha_1}{\chi^2\gamma_1 + \beta_1}},$$

where $\alpha_1(a^2\alpha_1 + b) > 0$ and $\alpha_1(\chi^2\gamma_1 + \beta_1) < 0$.
In this result, we deduce the periodic solutions:

$$u(x,t) = \sqrt{-\frac{a^2\alpha_1 + b}{\chi^2\gamma_1 + \beta_1}}\tan\left(\sqrt{\frac{a^2\alpha_1 + b}{2\alpha_1}}\eta\right)e^{i[-ax+bt+c_0+\sigma W(t)]}, \quad (58)$$

$$v(x,t) = \Theta\left\{\sqrt{-\frac{a^2\alpha_1 + b}{\chi^2\gamma_1 + \beta_1}}\tan\left(\sqrt{\frac{a^2\alpha_1 + b}{2\alpha_1}}\eta\right)\right\}e^{i[-ax+bt+c_0+\sigma W(t)]}, \quad (59)$$

and

$$u(x,t) = -\sqrt{-\frac{a^2\alpha_1 + b}{\chi^2\gamma_1 + \beta_1}}\cot\left(\sqrt{\frac{a^2\alpha_1 + b}{2\alpha_1}}\eta\right)e^{i[-ax+bt+c_0+\sigma W(t)]}, \quad (60)$$

$$v(x,t) = \Theta\left\{-\sqrt{-\frac{a^2\alpha_1 + b}{\chi^2\gamma_1 + \beta_1}}\cot\left(\sqrt{\frac{a^2\alpha_1 + b}{2\alpha_1}}\eta\right)\right\}e^{i[-ax+bt+c_0+\sigma W(t)]}, \quad (61)$$

provided $(a^2\alpha_1 + b)(\chi^2\gamma_1 + \beta_1) < 0$.

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Result 3.

$$\Upsilon = \frac{2(a^2\alpha_1 + b)}{\alpha_1},\ \mu = \mu,\ A_0 = 0,$$
$$A_1 = \sqrt{\frac{\alpha_1^2(\mu^2 - 1)}{4(\chi^2\gamma_1 + \beta_1)(a^2\alpha_1 + b)}},\quad B_1 = \sqrt{-\frac{\alpha_1}{2(\chi^2\gamma_1 + \beta_1)}}, \tag{62}$$

where $\alpha_1(a^2\alpha_1 + b) > 0$ and $\alpha_1(\chi^2\gamma_1 + \beta_1) < 0$.
From (62), we obtain the periodic solutions:

$$u(x,t) = \left\{\sqrt{\frac{(\mu^2-1)(a^2\alpha_1+b)}{(\chi^2\gamma_1+\beta_1)}}\left(\frac{\sec\left(\sqrt{\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)}{\mu\sec\left(\sqrt{\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)+1}\right) + \sqrt{-\frac{a^2\alpha_1+b}{\chi^2\gamma_1+\beta_1}}\left(\frac{\tan\left(\sqrt{\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)}{\mu\sec\left(\sqrt{\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)+1}\right)\right\}e^{i[-ax+bt+c_0+\sigma W(t)]}, \tag{63}$$

$$v(x,t) = \Theta\left\{\sqrt{\frac{(\mu^2-1)(a^2\alpha_1+b)}{(\chi^2\gamma_1+\beta_1)}}\left(\frac{\sec\left(\sqrt{\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)}{\mu\sec\left(\sqrt{\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)+1}\right) + \sqrt{-\frac{a^2\alpha_1+b}{\chi^2\gamma_1+\beta_1}}\left(\frac{\tan\left(\sqrt{\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)}{\mu\sec\left(\sqrt{\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)+1}\right)\right\}e^{i[-ax+bt+c_0+\sigma W(t)]}, \tag{64}$$

and

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$$u(x,t)=\left\{\sqrt{\frac{(\mu^2-1)(a^2\alpha_1+b)}{(\chi^2\gamma_1+\beta_1)}}\left(\frac{\csc\left(\sqrt{\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)}{\mu\csc\left(\sqrt{\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)+1}\right)-\sqrt{-\frac{a^2\alpha_1+b}{\chi^2\gamma_1+\beta_1}}\left(\frac{\cot\left(\sqrt{\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)}{\mu\csc\left(\sqrt{\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)+1}\right)\right\}e^{i[-ax+bt+c_0+\sigma W(t)]}, \tag{65}$$

$$v(x,t)=\Theta\left\{\sqrt{\frac{(\mu^2-1)(a^2\alpha_1+b)}{(\chi^2\gamma_1+\beta_1)}}\left(\frac{\csc\left(\sqrt{\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)}{\mu\csc\left(\sqrt{\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)+1}\right)-\sqrt{-\frac{a^2\alpha_1+b}{\chi^2\gamma_1+\beta_1}}\left(\frac{\cot\left(\sqrt{\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)}{\mu\csc\left(\sqrt{\frac{2(a^2\alpha_1+b)}{\alpha_1}}\eta\right)+1}\right)\right\}e^{i[-ax+bt+c_0+\sigma W(t)]}, \tag{66}$$

Provided $(\mu^2-1)(a^2\alpha_1+b)(\chi^2\gamma_1+\beta_1)>0$.

Case 4. Putting $\Upsilon=\mu=0$, equation (13) has the formal solution:

$$U(\eta)=A_0+A_1\tau(\eta), \tag{67}$$

Where A_0 and A_1 are constants such that $A_1\neq 0$ where $\tau(\eta)$ satisfies the non-linear ODE (25):
substituting (67) along with (25) into equation (13), collecting the coefficients of each power $\tau^i(\eta)$, $(i=0,1,2,3)$ and setting these coefficients to zero, we have the following algebraic equations as follows:

$$\tau^3 \;:\; \chi^2A_1^3\gamma_1+A_1^3\beta_1+2A_1\alpha_1=0,$$

$$\tau^2 \;:\; 3\chi^2A_0A_1^2\gamma_1+3A_0A_1^2\beta_1=0,$$

$$\tau \;:\; 3\chi^2A_0^2A_1\gamma_1-a^2A_1\alpha_1+3A_0^2A_1\beta_1-bA_1=0,$$

$$\tau^0 \;:\; A_0^3\chi^2\gamma_1-A_0a^2\alpha_1+A_0^3\beta_1-A_0b=0.$$

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On solving the above algebraic equations via Maple software, we get

$$b = -a^2\alpha_1,\ A_0 = 0,\ A_1 = \sqrt{-\frac{2\alpha_1}{\chi^2\gamma_1 + \beta_1}}, \qquad (68)$$

Where $\alpha_1(\chi^2\gamma_1 + \beta_1) < 0$.
From (68), we reach rational wave solutions:

$$u(x,t) = \left\{\sqrt{-\frac{2\alpha_1}{\chi^2\gamma_1 + \beta_1}}\left(\frac{1}{\eta}\right)\right\} e^{i[-ax+bt+c_0+\sigma W(t)]}, \qquad (69)$$

$$v(x,t) = \Theta\left\{\sqrt{-\frac{2\alpha_1}{\chi^2\gamma_1 + \beta_1}}\left(\frac{1}{\eta}\right)\right\} e^{i[-ax+bt+c_0+\sigma W(t)]}, \qquad (70)$$

Where $\eta = x + 2a\alpha_1 t$.

6. Conclusions

In this paper, the generalized projective Riccati equations scheme has been employed to discover solitons and other exact solutions of the coupled one-dimensional perturbed N-LSE with Kerr-law nonlinearity and multiplicative white noise. This study has presented bright, dark, singular solitons, in addition to solutions for rational and periodic functions, reported for the first time. The proposed scheme demonstrates effectiveness and can be utilized in multiple applications of different non-linear PDEs. By comparing the solutions obtained in this paper using the GPREs scheme with the solutions provided in [7], we noted that all our solutions are different from them and that each solution is a new one. Finally, all solutions obtained in this paper have been verified using Maple 2023 by returning them to the fundamental equations (2) and (3).

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