

## Development of Effective Modulus of Elasticity for an Eco-Concrete Based on Glass Powder.

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**P.H.D. Abdelmoutalib BENFRID**  
(ORCID: 0009-0007-8171-1654)  
*LSMAGCTP/ FT/ UDL-SBA, Algeria*  
*ATRST, Algeria*

**Email:** [benfridabdelmoutalib2050@gmail.com](mailto:benfridabdelmoutalib2050@gmail.com)

+213-696939173

**Pr. Mohammed BACHIR BOUIADJRA**  
(ORCID: 0009-0008-4814-6187)  
*LSMAGCTP/ FT/ UDL-SBA, Algeria*  
*ATRST, Algeria*

**Email:** [mohamedbachirbouiadjra@gmail.com](mailto:mohamedbachirbouiadjra@gmail.com)

+213-661200447

### Abstract:

This study aims to develop an effective modulus of elasticity for eco-concrete using glass powder. Various theoretical models, including VOIGT and REUSS-VOIGT, are compared with experimental results. The REUSS-VOIGT model proved most accurate, particularly for cement replacement (5%-20%) and fine aggregate replacement (up to 25%). The REUSS-VOIGT model closely matches experimental results, especially after 56 days. To achieving sustainable development in the production of Portland cement by conserving natural resources and rationalizing their consumption for future generations.

**Keywords:** Homogenization, Effective Modulus of Elasticity, Eco-concrete, Glass Powder, REUSS/VOIGT Model.

## تطوير معامل المرونة الفعال للخرسانة الصديقة للبيئة القائمة على مسحوق الزجاج

عبد المطلب بن فريد و محمد بشير بويجرة  
الجزائر

### الملخص:

يهدف هذا العمل إلى تطوير معامل مرونة فعال للخرسانة الصديقة للبيئة باستخدام مسحوق الزجاج. تتم مقارنة نماذج نظرية مختلفة، بما في ذلك نماذج VOIGT و REUSS-VOIGT، مع النتائج التجريبية. أثبت نموذج REUSS-VOIGT أنه الأكثر دقة، خاصة عند استبدال الأسمنت بنسبة (5% - 20%) واستبدال الركام الناعم بنسبة تصل إلى 25%. يتوافق نموذج REUSS-VOIGT بشكل وثيق مع النتائج التجريبية، خاصة بعد 56 يوماً. بغية تحقيق تنمية مستدامة في إنتاج الإسمنت البورتلندي من خلال الحفاظ على الموارد الطبيعية وترشيد استهلاكها للأجيال القادمة.

**الكلمات المفتاحية:** تجانس، معامل المرونة الفعال، الخرسانة الصديقة للبيئة، مسحوق الزجاج، نموذج REUSS/VOIGT.

### Notations:

**Em:** The modulus of elasticity of concrete.

**Eg:** The modulus of elasticity of glass powder.

**Ep:** The modulus of elasticity of the parallel component determined by the VOIGT model.

**Es:** The modulus of elasticity of the series component determined by the REUSS model.

**Ev:** The modulus of elasticity determined by the VOIGT model.

**Er:** The modulus of elasticity determined by the REUSS model.

**Evr:** The modulus of elasticity determined by the VOIGT/REUSS model.

**Erv:** The modulus of elasticity determined by the REUSS/VOIGT model.

**Eij:** The modulus of elasticity of concrete obtained using the BAEL formula.

**Eexp:** The modulus of elasticity of concrete obtained through experiments.

**V:** The total volume of the mix.

**Vm:** The volume of concrete.

**Vg:** The volume of glass powder.

**P.V:** The percentage of glass powder.

**Fcj:** The compressive strength of concrete at age ( $j$ ) days.

### Introduction

In the civil engineering literature, several experimental studies have been conducted on glass powder-based concrete. However, scientific reviews that establish homogenization rules are less commonly found. Mansouri Khelifa et al., in their doctoral thesis in 2021, provided the following parameters for glass powder particles [1]:

$E_g = 64 \text{ GPa}$  (Modulus of Elasticity of glass powder)

$\nu_g = 0.2$  (Poisson's Ratio)

$\rho_g = 2.54 \text{ g/cm}^3$  (Density)

For concrete incorporating glass powder, the following parameters are used for comparison:

$E_m = E_{ij}$  (reference 0% glass powder) or  $E_{exp}$  (reference 0% glass powder) GPa (Modulus of Elasticity of concrete)

$\nu_m = 0.3$  (Poisson's Ratio)

$\rho_m = 2.5 \text{ kg/m}^3$  (Density)

The Voigt model, the Reuss model, the Voigt-Reuss model, and the Reuss-Voigt model are well-defined homogenization models in reference [2]. The BAEL (1991) code [3] is the most commonly used reference for the calculation of reinforced concrete in North Africa.

The use of glass powder in concrete has garnered increasing interest due to its potential to enhance properties and contribute to more sustainable construction practices.

In 2014, Shilpa Raju and Kumar studied concrete incorporating glass powder with a diameter of  $45 \mu\text{m}$ . They replaced a portion of the cement with glass powder and conducted mechanical tests on the concrete samples. They observed that the compressive strength of the concrete increased with the addition of glass powder up to 20%, indicating that glass powder can effectively replace a portion of the cement [4]. In 2019, Zhi-hai He et al. investigated three properties—compressive strength, modulus of elasticity, and creep of concrete—by varying the glass powder content between 0% and 30%, which partially replaces cement. They found that using glass powder up to 20% provides the optimal compressive strength, improves the modulus of elasticity, and effectively reduces creep [5]. In 2020, Zhi-hai He et al. conducted several comparisons with existing literature on glass powder. They observed that the mechanical properties of concrete incorporating glass powder are influenced by factors such as the type and color of the glass, the particle size of the glass powder, and the silica content of the glass [6]. In 2017, Sudhanshu Kumar and Bharat Nagar prepared concrete incorporating glass powder, adding 5% of the powder to replace fine aggregates and comparing it with a reference M20 concrete (0% glass powder). They evaluated the compressive strength at 3, 7, and 28 days. Their experiments demonstrated that up to 25% of the fine aggregates can be replaced by glass powder without compromising strength, but beyond this threshold, a noticeable decrease in

resistance is observed [7]. In 2015, Richard Morin et al. used glass powder concrete to construct a sidewalk in Montreal and conducted ongoing monitoring of this material. They found that the mechanical properties met the city's requirements. The results of the scaling tests performed on the reference slabs maintained under real climatic conditions were valid for 10% and 20% glass powder [8]. In 2022, Aadil Manzoor et al. conducted a similar study by analyzing the mechanical behavior of concrete with a partial replacement of cement with crushed glass waste (glass powder). They concluded that it is possible to achieve up to a 15% replacement of cement with glass powder [9]. In 2021, Shriram et al. conducted a study on concrete mixtures using various powders (slag, fly ash, and alkali-activated glass). They tested the modulus of elasticity of concrete by partially replacing cement with glass powder and found that this modulus reaches its maximum value at a 15% glass powder replacement level [10].

All the previously discussed works are experimental studies. Now, we will describe some studies guided by homogenization and theories that approach practical reality.

In 2022, CHATBI et al. examined how silicon dioxide ( $\text{SiO}_2$ ) nanoparticles affect the static behavior of concrete beams. Utilizing Voigt's model for nanoparticle agglomeration and higher-order shear deformation theory, they found that  $\text{SiO}_2$  nanoparticles enhance mechanical resistance, reduce deflections and stresses, and that the elastic foundation significantly impacts beam bending. Both analytical and numerical methods were used to evaluate various parameters affecting beam performance [11]. In 2021, Harrat et al. investigated the incorporation of  $\text{SiO}_2$  nanoparticles in concrete plates, showing that they improve mechanical performance. Using Voigt's model and higher-order shear deformation theory, they modeled the plates on a Pasternak elastic foundation, deriving equilibrium and energy equations with virtual work and Hamilton's principle. The results indicated that optimal  $\text{SiO}_2$  nanoparticle levels enhance mechanical behavior, and the elastic foundation plays a crucial role in slab bending [12]. In 2023, Benfrid et al.

focused on the thermomechanical effects of glass powder in concrete. They employed Eshelby's model and higher-order shear deformation theory for simulations, deriving equilibrium and energy equations through virtual work and Hamilton's principle. The study, using Navier's techniques for closed-form solutions, revealed challenges in using glass powder for thermomechanical applications and provided new guidelines for future research on reinforced concrete [13]. In 2024, Benfrid et al. explored thermal buckling in concrete slabs with homogenized crushed bovine bone particles. They aimed to determine the maximum temperature for thermal buckling in these modified slabs. This study not only investigates the thermal properties of concrete with bone powder but also emphasizes environmental and economic benefits, such as recycling cow bones, reducing waste, and minimizing environmental damage from traditional sand mining. The research points to the potential for creating sustainable, eco-friendly concrete materials and advancing waste recycling in construction [14].

In this work, the objective is to develop a theoretical homogenization model for eco-concrete based on glass powder. The approach involves comparing the results with several experimental studies on eco-concretes or concretes containing glass powder. If the concrete's elasticity modulus is known ( $E_{exp}$ ), the reference concrete (0% glass powder) is used as the matrix ( $E_m$ ), and the glass powder as the reinforcement ( $E_g$ ). When the elasticity modulus is not specified, we calculate ( $E_{ij}$ ) using the BAEL 91 regulations, with the reference concrete ( $E_{ij}$ ) at 0% glass powder as the baseline.

We gradually increase the proportion of glass powder up to 20%, as experiments show that the maximum strength ( $F_c \max$ ) of concrete containing glass powder is limited to 20% when the powder replaces cement, and up to 25% when it replaces fine aggregates.

This study principal project is part of energy conservation efforts as it reduces the mechanical work required in cement production and benefits water in the humid environment of cement plants. Additionally, this technique reduces the emission of toxic gases. In terms of environmental resource conservation, replacing 25% of

cement with glass powder can offer a promising future for limestone and clay deposits, which are used as raw materials in cement production.

### Homogenization models

Considering that eco-concrete is a bi-phase material, with reference concrete as the matrix and glass powder as the reinforcement.

Knowing that:

$$V_g + V_m = V \quad (1)$$

- REUSS [2]:

$$\frac{1}{E_r} = \frac{1-V_g}{V \cdot E_m} + \frac{V_g}{V \cdot E_g} \quad (2)$$

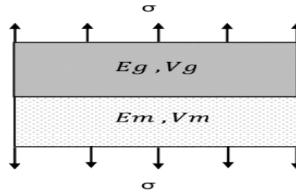


Figure 01. REUSS Model.

- Voigt [2]:

$$E_v = \frac{E_g \cdot V_g}{V} + \frac{E_m \cdot (1-V_g)}{V} \quad (3)$$

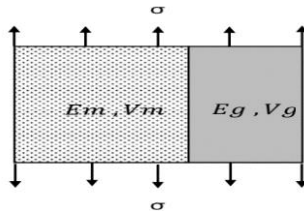
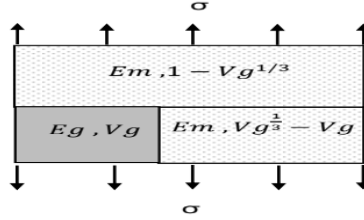


Figure 02.VOIGT Model.

- **Voigt-REUSS [2] :**

$$E_p = E_m + V_g \frac{2}{3} (E_g - E_m) \quad (4)$$

$$\frac{1}{E_{vr}} = \frac{1}{E_m} + V_g \frac{1}{3} \left( \frac{1}{E_p} - \frac{1}{E_m} \right) \quad (5)$$

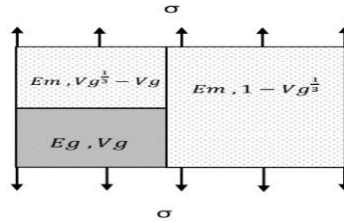


**Figure 03.** VOIGT/REUSS Model.

- **REUSS-Voigt[2] :**

$$\frac{1}{E_s} = \frac{1}{E_m} + V_g \frac{2}{3} \left( \frac{1}{E_g} - \frac{1}{E_m} \right) \quad (6)$$

$$E_{rv} = E_m + V_g \frac{1}{3} (E_s - E_m) \quad (7)$$



**Figure 04.** REUSS/VOIGT Model.

- **BAEL ( Reinforced concrete at limit states )**

The BAEL (1991) rule [3] defines the following two modules:

$$E_{ij} = 11000 * f_{cj}^{1/3} \quad (\text{MPa}) \quad (8)$$

$$E_{vj} = \frac{E_{ij}}{3} \quad (\text{MPa}) \quad (9)$$



## Results and comparisons

Here, we present the results shown in the following tables, which compare theory and experimental data.

**Table 01. Comparison between Eeff and Eij obtained from the results of (Shilpa Raju and Kumar) (2014) at the age of 28 days.**

P.V	0%	5%	10%	15%	20%	25%	30%	35%	40%
Fc28	27.05	28.58	29.77	31.56	33.5	30.52	24.22	22.44	19.03
Eij	33.02	33.63	34.09	34.76	35.46	34.38	31.83	31.03	29.37
Er	33.02	33.84	34.70	35.61	36.56	37.57	38.63	39.76	40.95
Ev	33.02	34.57	36.12	37.67	39.22	40.77	42.31	43.86	45.41
Evr	33.02	34.45	35.82	37.15	38.49	39.83	41.18	42.55	43.95
Erv	33.02	33.21	33.60	34.12	34.77	35.53	36.40	37.38	38.48
$\Delta(Erv - Eij)$	0	-0.42	-0.49	-0.64	-0.68	-	-	-	-

**Table 02. Comparison between Eeff and Eij obtained from the results of (Shilpa Raju and Kumar) (2014) at the age of 90 days.**

P.V	0%	5%	10%	15%	20%	25%	30%	35%	40%
Fc90	27.33	28.87	30.08	31.85	33.86	30.82	24.44	22.72	19.25
Eij	33.13	33.74	34.21	34.87	35.59	34.49	31.92	31.16	29.48
Er	33.13	33.95	34.81	35.71	36.67	37.67	38.74	39.86	41.05
Ev	33.13	34.67	36.22	37.76	39.30	40.85	42.39	43.93	45.48
Evr	33.13	34.56	35.92	37.25	38.58	39.92	41.27	42.64	44.03
Erv	33.13	33.32	33.70	34.23	34.88	35.63	36.50	37.48	38.57
$\Delta(Erv - Eij)$	0	-0.42	-0.51	-0.64	-0.71	-	-	-	-

**Table 03. Comparison between Eeff and Eij obtained from the results of (Zhi-hai He et al ...) (2019) at the age of 28 days.**

P.V	0%	10%	20%	30%
Fc28	57.5	55	52.5	41
Eij	42.46	41.83	41.19	37.93
Er	42.46	43.94	45.52	47.23
Ev	42.46	44.61	46.77	48.92
Evr	42.46	44.49	46.48	48.47
Erv	42.46	42.89	43.78	45.03
$\Delta(Erv - Eij)$	0	+1.06	+2.6	-

**Table 04. Comparison between Eeff and Eij obtained from the results of (Zhi-hai He et al) (2019) at the age of 90 days.**

P.V	0%	10%	20%	30%
Fc 90 days	63	65	73	54
Eij	43.77	44.23	45.97	41.58
Er	43.77	45.20	46.72	48.36
Ev	43.77	45.79	47.82	49.84
Evr	43.77	45.69	47.57	49.45
Erv	43.77	44.17	45.02	46.21
$\Delta(Erv - Eij)$	0	-0.06	-0.95	

**Table 05. Comparison between Eeff and Eexp obtained from the results of (Zhi-hai He et al) (2019) at the age of 28 days.**

P.V	0%	10%	20%	30%
Eexp	35.00	33.00	32.50	29.50
Er	35.00	36.66	38.49	40.51
Ev	35.00	37.90	40.80	43.70
Evr	35.00	37.65	40.19	42.75
Erv	35.00	35.55	36.67	38.24
$\Delta(Erv - Eexp)$	0	+2.55	+4.17	

**Table 06. Comparison between Eeff and Eexp obtained from the results of (Zhi-hai He et al) (2019) at the age of 90 days.**

P.V	0%	10%	20%	30%
Eexp	36.00	36.25	36.50	33.00
Er	36.00	37.65	39.45	41.44
Ev	36.00	38.80	41.60	44.40
Evr	36.00	38.57	41.04	43.53
Erv	36.00	36.53	37.63	39.16
$\Delta(Erv - Eexp)$	0	+0.28	+1.13	

Table 07. Comparison between Eeff and Eij obtained from the results of (Richard Morin et al) (2015) at the age of 28 days.

P.V	0%	10%	20%	25%
Fc28	38.90	36.70	35.40	35.30
Eij	37.27	36.56	36.12	36.08
Er	37.27	38.89	40.67	41.62
Ev	37.27	39.94	42.62	43.95
Evr	37.27	39.74	42.12	43.32
Erv	37.27	37.78	38.84	39.53
$\Delta(Erv - Eij)$	0	+1.22	+2.72	+2.73

Table 08. Comparison between Eeff and Eij obtained from the results of (Richard Morin et al...) (2015) at the age of 90 days.

P.V	0%	10%	20%	25%
Fc 90 days	44.90	47.00	45.00	42.50
Eij	39.10	39.70	39.13	38.39
Er	39.10	40.68	42.40	43.31
Ev	39.10	41.59	44.08	45.33
Evr	39.10	41.42	43.67	44.79
Erv	39.10	39.58	40.59	41.24
$\Delta(Erv - Eij)$	0	-0.12	+1.46	+2.87

Table 09. Comparison between Eeff and Eij obtained from the results of (Richard Morin et al) (2015) at the age of 1 year.

P.V	0%	10%	20%
Fc 1 year	45.40	52.60	47.60
Eij	39.24	41.21	39.87
Er	39.24	40.82	42.53
Ev	39.24	41.72	44.19
Evr	39.24	41.55	43.78
Erv	39.24	39.72	40.72
Eexp	38.4	43.1	43.1
$\Delta(Erv - Eij)$	0	+1.89	+3.23

**Table 10. Comparison between Eeff and Eij obtained from the results of (Aadil Manzoor et al.) (2022) at the age of 28 days.**

P.V	0%	5%	10%	15%
Fc	43.81	41.57	39.25	44.54
Eij	38.78	38.11	37.38	38.99
Er	38.78	39.56	40.37	41.22
Ev	38.78	40.04	41.30	42.56
Evr	38.78	39.97	41.13	42.26
Erv	38.78	38.94	39.27	39.72
$\Delta(Erv - Eij)$	0	+0.83	+1.89	+0.73

**Table 11. Comparison between Eeff and Eij obtained from the results of (Aadil Manzoor et al.) (2022) at the age of 56 days.**

P.V	0%	5%	10%	15%
Fc	42.23	44.63	45.89	47.24
Eij	38.31	39.02	39.38	39.76
Er	38.31	39.09	39.91	40.76
Ev	38.31	39.59	40.88	42.16
Evr	38.31	39.52	40.69	41.85
Erv	38.31	38.47	38.81	39.27
$\Delta(Erv - Eij)$	0	-0.55	-0.57	-0.49

**Table 12. Comparison between Eeff and Eij obtained from the results of (Aadil Manzoor et al.) (2022) at the age of 90 days.**

P.V	0%	5%	10%	15%
Fc	51.25	52.67	53.12	51.47
Eij	40.86	41.23	41.35	40.92
Er	40.86	41.61	42.39	43.20
Ev	40.86	42.02	43.17	44.33
Evr	40.86	41.96	43.03	44.09
Erv	40.86	41.01	41.31	41.74
$\Delta(Erv - Eij)$	0	-0.22	-0.04	+0.82

**Table 13. Comparison between Eeff and Eexp obtained from the results of (Aadil Manzoor et al ...) (2022) at the age of 90 days.**

P.V	0%	5%	10%	15%
Eexp	33.10	32.30	33.10	34.20
Er	33.10	33.92	34.78	35.68
Ev	33.10	34.65	36.19	37.74
Evr	33.10	34.53	35.89	37.23
Erv	33.10	33.29	33.67	34.20
$\Delta(Erv - Eij)$	0	+0.99	+0.57	0

### Discussion of the results

1. Replacing a portion of cement with glass powder of diameter (45  $\mu\text{m}$ ): The results obtained for the calculated modulus of elasticity are very close to the RUESS/VOIGT modulus within the range of [5%-20%] glass powder, as shown in Comparison No. 01.
2. When glass powder replaces cement with a very similar particle size: The RUESS/VOIGT model is closer in the long term to Eij [Tables 4-6] as shown in Comparison No. 02.
3. Replacing fine aggregates with glass powder: It also shows that the most accurate theoretical homogenization model is the RUESS/VOIGT model, but there is the possibility of going up to 25% glass powder, as observed in Comparison No. 03.
4. Comparison No. 04 reinforces the idea from Comparison No. 02: That glass powder in concrete performs well in the long term, and the modulus of elasticity calculated using the BAEL formula is closer to the effective modulus of RUESS/VOIGT, but limited to 20%.
5. In the long term: The experimental modulus of elasticity remains close to the RUESS/VOIGT model, as presented in Comparison No. 05.

6. For alkali-active glasses: It is noted that 15% glass powder as a cement replacement introduces slight uncertainty between  $E_{exp}$  and  $E_{rv}$ , with this uncertainty being highlighted in Comparison No. 06.

## Conclusion

We can conclude the following points:

- The most accurate model for homogenizing eco-concrete based on glass powder is the RUESS/VOIGT model.
- This model is valid for eco-concrete based on glass powder over the long term (+56 days), as the strength of concrete with glass powder improves over time.
- The RUESS/VOIGT model is applicable up to 20% glass powder if it replaces part of the cement and up to 25% glass powder if it replaces fine aggregates.
- The maximum uncertainty between the experimental results and this theoretical model is 3.23 GPa.
- Maintain energy efficiency and achieve a 30% reduction in energy consumption, including during the startup of cement plants. Minimize 30% of the heat emitted by cement plants, which provides an environmental benefit.
- Preserve limestone and clay deposits for future generations and recycle 30% of glass waste. Reduce toxic gases and reactive powders that cause serious dermatological and pulmonary diseases
- Improve the energy field with this technique to reinforce the linings and highlight the solidification of well and petroleum reservoir walls.

- Enhance the scientific literature to effectively predict the results of pozzolanic materials and forecast the mechanical behavior of concrete before conducting experiments and awaiting extended durations.
- Glass powder provides effective durability to concrete, making it suitable for constructing storage tanks for materials and liquids, including those used for storing or conserving petroleum and materials involved in the preparation of petrochemical products. Glass powder is used with liquid polymers as a mixture to clean oil distribution pipes. It is preferable to use this technique in reservoirs and water tanks, as glass powder provides exceptional impermeability.

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