

## Modeling and Design of Liquid Level Control System

Mahmoud N. Zaggout    Ismail M. Albatrookh    Omar A. Essaid

Faculty of Engineering,  
Misurata University,  
Misurata, Libya.

Faculty of Engineering,  
Misurata University,  
Misurata, Libya

Higher Institute  
for sciences &  
Technology,  
Misurata, Libya.

[m.zaggout@eng.misuratau.edu.ly](mailto:m.zaggout@eng.misuratau.edu.ly)

### Abstract

The industrial applications of a single tank system are widely used especially in the food process industries. A control of liquid level in a tank is a problem in the process technologies. The accurate control of liquid level effects on the product quality and efficiency, as well as its effect on the safety of the production process. This work proposes the digital controller to maintain and regulate a desired liquid level in the tank. A mathematical model of the single tank system was built in the MATLAB Simulink. The control design method is demonstrated and verified in simulation and experimental environments. Based on the performance of the system in terms of desired liquid level, rise time, settling time and steady state error, the results show that the proposed method of the digital control design gives simple and effective controller for liquid level system. This technique can be easily embedded in the real tank system.

**Keywords:** Tank system, Digital control, and P-controller

## نمذجة وتصميم نظام تحكم بمستوى سائل

محمود نوح زقوط<sup>1</sup>، إسماعيل محمد البطروخ<sup>1</sup>، عمر أبوبكر الصيد<sup>2</sup>

1. كلية الهندسة، جامعة مصراتة، مصراتة، ليبيا.
2. المعهد العالي للعلوم والتقنية، مصراتة، ليبيا.

m.zaggout@eng.misuratau.edu.ly

### الملخص

يستخدم نظام الخزان الأحادي على نطاق واسع في التطبيقات الصناعية خاصة في الصناعات الغذائية، حيث يعد التحكم في مستوى السائل بهذا الخزان من أكبر المشاكل من الناحية العملية. يؤثر التحكم الدقيق في مستوى السائل على جودة المنتجات وكفاءتها، فضلاً عن تأثيره على عملية الإنتاج وسلامتها. يقترح هذا العمل متحكم رقمي للحفاظ على مستوى السائل المطلوب في الخزان وتنظيمه. تناولت الدراسة بناء نموذج رياضي لنظام خزان أحادي حقيقي، وتوضيح خطوات تصميم المتحكم والتحقق منها باستخدام برامج المحاكاة ماتلاب والتجارب العملية. بناءً على أداء النظام من حيث ارتفاع مستوى السائل المطلوب، زمن الصعود، زمن الوصول للحالة المستقرة، وخطأ الحالة المستقرة. أظهرت النتائج أن الطريقة المقترحة لتصميم المتحكم الرقمي توفر تحكماً بسيطاً وفعالاً لمستوى سائل الخزان. هذه التقنية يمكن تضمينها بسهولة لتصميم المتحكمات في أنظمة الخزانات العملية.

**الكلمات المفتاحية:** نظام خزان، تحكم رقمي، متحكم تناسبي.

### 1. Introduction

In industry, there are generally four types of controlled quantities: liquid level, temperature, pressure, and flow rate. The control technique of the liquid level has become a common control strategy in most current industrial production, and its impact on production

cannot be ignored [1-2]. Therefore, the research on liquid level control has practical significance and broad application prospects. For over the last 30 to 40 years, intensive research has been published in controlling the liquid level of the single tank systems. Work including [1-5] focused on using the analog P, PI, PD and PID controllers, as well as fuzzy controller. The work also expanded into the coupled tank systems [6-7], and three tank systems [8]. Other control methods have been proposed for the liquid level control problem using a neural network based PID controller [9-10].

Although all proposed controllers have proven successful, applying them in industry process is difficult, since the current control process have been achieved through computers and PLCs where the sampling time has an effect. Therefore, the digital controller should be used instead of analog controller.

The work described in this paper aims to design a simple digital controller for a single tank liquid level system, based on time response specifications. The influence of the sampling time on the control parameters, are presented and the control effectiveness is demonstrated by simulation and verified on a realistic tank system. The rest of the paper is organized as follows: Section 2 describes the overall single tank system and its mathematical model. Section 3, presents the proposed controller, and design steps. Section 4 shows the simulation and experimental results for the proposed controller. These results will demonstrate that the proposed method can be simply and reliably used. Some conclusions are drawn in Section 5.

## 2. Single Tank System

### 2.1 System Description

The system is presented in this study is specifically designed to control the yogurt level in the tank, which is located on the top of the cup machine filler in a yogurt factory. The maximum production capacity of the machine is up to  $80000 \text{ cup/hr}$ . The main purpose of the system is controlling the flow rate, which the pump delivers yogurt to the tank and so it can reach the desired level inside the

tank. The whole system is represented in figure1. The crucial components of this system are:

- Source: it is a large tank and used to keep the yogurt after mixing process and before production process.
- Tank: it is a small tank and used to hold the desired amount of yogurt for production.
- Pump: the yogurt from the source is fed to the tank through the pump when actuated by the controller.
- Sensor: senses the level of the yogurt inside the tank.
- PLC: it holds the digital controller, which is used to maintain the desired level by starting and stopping the pump when it receives information from the sensor as well as opening and closing the valve.

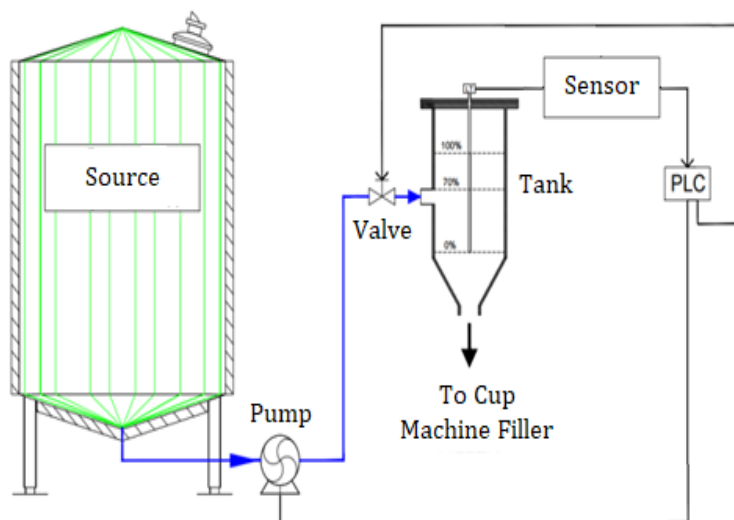


Figure 1. Schematic diagram of a yogurt level control system

## 2.2 Mathematical Model

In this part, both the storage tank and the valve, in figure 1, were neglected because they have no effect on the control process. Meanwhile, the mathematical model of the other elements was carried out as follows:

### a. Tank

The mathematical modeling of liquid level in the tank is obtained using mass balance. In this system, the liquid is flowing to the tank at flow rate  $q_{in}$  and the outlet flow is  $q_o$ . Note that  $q_o$  is a constant and depends on the production process. The cross-section area of tank is  $A$ . The height of the liquid level in the tank is represented by  $h$  which is controlled by adjusting the flowrate of the pump. Assuming the density of the inlet and outlet flow rate is constant and the tank has a uniform cross-sectional area [3].

$$A \frac{dh}{dt} = q_{in} - q_o \quad (1)$$

By taking Laplace transform of equation (1) and all initial conditions equal zero, it becomes

$$AsH(s) = Q_{in}(s) - Q_o(s) \quad (2)$$

For this system, there are two cases which are:

#### Case I: No production process

In this case  $q_o = 0$  and the transfer function of the system is

$$\frac{H(s)}{Q_{in}(s)} = \frac{1}{As} \quad (3)$$

#### Case II: With production process

Equation (2) can be rewritten as following

$$\frac{H(s)}{Q_{in}(s) - Q_o(s)} = \frac{1}{As} \quad (4)$$

### b. Digital Controller

It is built inside the PLC and consists from the main three parts that connected in series as following:

- Sampler, to convert the continuous signal to discrete signal with sampling time  $T$ .

- Controller: it is usually a PID controller with ability to modification to P, PI, PD controllers as desired by the operator. The transfer function of PID controller  $G_c(s)$  is

$$G_c(s) = K_p \left( 1 + \frac{1}{T_i s} + T_d s \right) \quad (5)$$

where  $K_p$ ,  $T_i$  and  $T_d$  are the proportional gain, the integral time and the derivative time respectively.

- Zero Order Hold Circuit (ZOH): to convert the output discrete control signal to a continuous signal. The transfer function of the circuit  $G_{zoh}(s)$  is given by

$$G_{zoh}(s) = \frac{1 - e^{-Ts}}{s} \quad (6)$$

### c. Pump

The mathematical model of the pump is simplified into only a gain, and then its transfer function becomes

$$G_{pu}(s) = K_{pu} \quad (7)$$

where  $K_{pu}$  is the sensor's gain.

### d. Sensor

It measures the liquid level in the tank and converts it into electrical signal where the PLC can deal with it. Therefore, with neglecting its time delay, the transfer function of the sensor  $G_s(s)$  can be represented by

$$G_s(s) = K_s \quad (8)$$

where  $K_s$  is the sensor's gain. The block diagram of the mathematical model of the whole system is represented in figure 2.

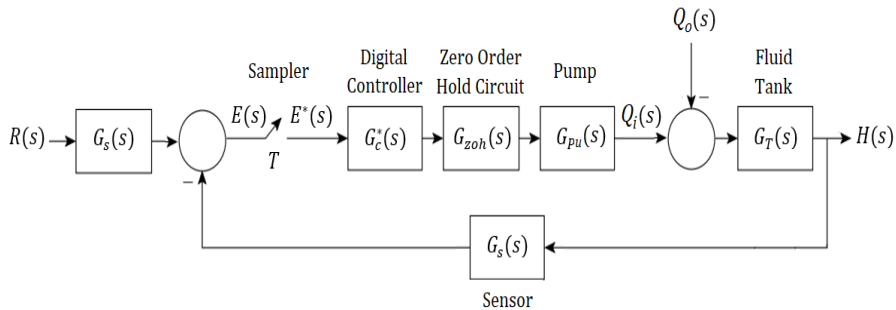


Figure 2. Block diagram of the liquid level control system

### 3. Controller Design

The design process depends on the desired system response, and it starts with determining the controller type, and then calculating the values of the controller's parameters. In this work, the choice of the controller's type depends on two conditions, which are:

1. The steady-state error of the system ( $e_{SS}$ ) is equal zero.
2. The settling time of the system ( $t_s$ ) is within certain limits and without oscillations.

In order to choose the controller type, let assume the P-controller is used and evaluated for the steady state error condition with no production ( $q_o = 0$ ). With z-transform, the block diagram of the continuous system in figure 2 can be transformed into discrete system as shown in figure 3.

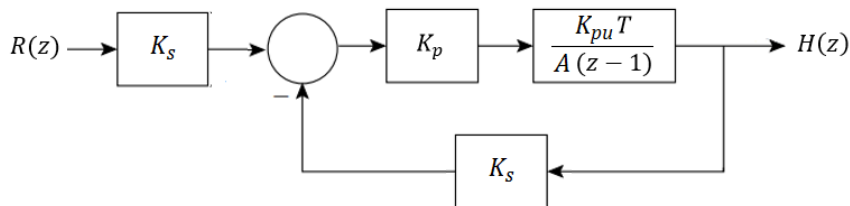


Figure 3. Block diagram of the discrete liquid level control system

The static position error constant  $k_p$  of the discrete system, in figure 3, as well as the steady state error can be calculated by

$$k_p = \lim_{z \rightarrow 1} \frac{K_p K_{pu} K_s T}{A(z-1)} = \frac{1}{0} = \infty \quad (9)$$

$$e_{ss} = \frac{1}{1+k_p} = \frac{1}{1+\infty} = 0 \quad (10)$$

It can be clearly seen that the steady state error can be easily eliminated with the P-controller, and therefore the P-controller is satisfied. For the second condition, the relation between the controller parameter and the settling time can be derivative to be:

$$K_p = \frac{A \left( e^{\frac{t_s T}{4}} - 1 \right)}{K_{pu} K_s T} \quad (11)$$

From the above equation, the controller's proportional gain can be simply designed based on the settling time and sampling time values. Furthermore, the range of  $K_p$  values for the system stability is determined as following:

$$0 < K_p < \frac{A2}{K_{pu} K_s T} \quad (12)$$

From equations (11) and (12), the  $K_p$  value is an inversely proportional to the sampling time.

#### 4. Results and Discussions

In order to verify the mathematical model and the proposed design method several tests were carried out on the simulation and experimental environments. As mentioned earlier in section 2.1, the used system in this study is the yogurt level control system and its parameter values are illustrated in Table 1.



Table 1. System parameter values

Parameter	Symbol	Value	Unit
Area of tank	$A$	706.86	$cm^2$
Height of tank	$L$	33	$cm$
Desired yogurt level in tank	No production	26.4 (80%)	$cm$
	With production	26.4 – 21.5 (80% – 65%)	
Sampling time	$T$	0.5	$msec$
Pump gain	$K_{pu}$	350	$cm^3/V$
Sensor gain	$K_s$	0.1	$V/cm$
Outlet flow rate	$q_o$	275	$cm^3/sec$

According to equation (12), the range of  $K_p$  values was calculated to be:

$$0 < K_p < 80784 \quad (13)$$

To choose the appropriate settling time of the yogurt level control tank system, there are two important conditions:

- The filling process should not be fast, so that foam forms in the yogurt tank, which is difficult to get rid of and affects the product quality.
- The filling process should not be slow, so that the tank does not empty during the production process, and ensures no defect in the filling (the yogurt containers are partially empty).

Practically, the acceptable time for filling the tank is 30sec to 60sec. Therefore, the P-controller was designed according to equation (11) with the settling time value of 40 sec, and the proportional gain found to be:

$$K_p = 2 \quad (14)$$

For a step input reference value of 26.4 (80% of the tank height), the design was tested and evaluated in a noise-free MATLAB Simulink model, as well as experimentally in the real tank system. The resulted responses are shown in figure 4. Note that the measured response was sampled at 0.5Hz. Compared with simulation results, The measured signal incorporates a noise which was caused experimentally by a small continuous grid frequency fluctuation around 50Hz, and factory site grid voltage unbalance which contributes to pump stator unbalance and noise. These noise sources were unavoidable in the measurements from the system.

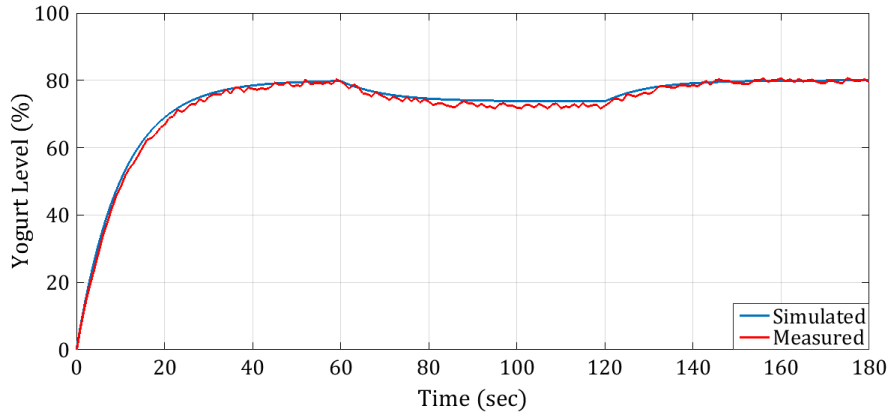


Figure 4. Simulated and measured responses

It can be easily seen the success of the P-control design to rise the yogurt level from 0% up 80% inside the tank in 40sec and maintained it at this level for the next 20sec. However, when the production process started at 60sec, the yogurt level decreased slightly and stabilized at 74% and 72% for the simulated and measured signals, respectively. From the system specifications, the desired yogurt level in the tank system is 65% – 80% during production process. When the production process was stopped at

120sec, the yogurt level started increasing again to reach 80%. Table 2 presents a comparison of the control design effectiveness in both simulation and experimental tests.

**Table 2. Comparison between simulated and measured results**

Parameter		Symbol	Simulation	Measurement
Settling time		$t_s$	40.0sec	43.5sec
Rise time		$t_r$	22.5sec	24.0sec
Steady state error	No production	$e_{ss}$	0.00cm	0.00cm
	With production		1.98cm	2.64cm

From the results, it can be observed that there is a small error in the steady state response within the production process, and it does not exceed 1.98cm(7.5%) and 2.64cm(10%) in the simulated and measured signals, respectively. This slight error is due to that the design was completed based on no production process. However, the yogurt level in the tank is still within the system specifications shown in the Table1. Therefore, the designed controller is succeeded in regulating the yogurt level inside the tank to the required level during or no production process.

Although, the results demonstrate the good agreement between simulated and measured signals from the model and real tank system, there are slight differences in the results. This difference is mainly due to the simple model used in the representation of the pump. Therefore, in order to obtain a more accurate model, the pump must be fully represented and included all its parts which are the pump, motor, gearbox, and inverter.

## 5. Conclusions

This paper presents a study on a modelling and control designing of liquid level system for single tank that used in private, chemical, industrial or other related applications. Digital P-Controller is implemented to control the yogurt level in tank within the desire

values. Simple design method was used to find controlling variables based on the settling time and sampling time is explained. The full mathematical model of the yogurt level control tank has been derived. The MATLAB simulation was used to test the controller's performance and then compare the results with those presented in the real tank system. From the results, both simulated and measured, the success of the controller design is proved where the desired performance of the height level in the tank and settling time are obtained. The designed technique is simple, attractive and could easily be used industry and other applications.

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### References

- [1] P. Jiang, "Summary of PID Control System of Liquid Level of a Single Capacity Tank," *Journal of Physics: Conference Series* 1865 022061 (2021).
- [2] S. Suda, "PID control theory and Practice" [M]. Taipei: Quan Hua Science and Technology Book Company (1992), 14 (3), pp. 23-26.
- [3] B. Getu, "Water Level Controlling System Using PID Controller," *International Journal of Applied Engineering Research* (2016), 11 (23), pp.11223-11227.
- [4] M. Dong, and B. Guo, "Research on Control Algorithm of Water-tank Water Level Control System," *International Journal of Control and Automation*(2016), 9(12), pp. 1-10.
- [5] S. Pratama, E. Susanto, and A. Wibowo, "Design and Implementation of Water Level Control Using Gain Scheduling PID Back Calculation Integrator Anti Windup," *The 2016 International Conference on Control, Electronics, Renewable Energy and Communications (ICCEREC)* (2016).
- [6] M. Fellani, and A. Gabaj, "PID Controller Design for Two Tanks Liquid Level Control System using Matlab," *International*

- Journal of Electrical and Computer Engineering (IJECE)(2015), 5 (3), pp. 436-442.
- [7] J. Ye, X. Zhang, C. Lv,P. Wang, and H. Lv, "Design of liquid level control system for double tank, "IOP Conference Series: Materials Science and Engineering 740 (2020) 012097(2020).
- [8] L. Chen, "Principle and Simulation PID Controller of Liquid Level System," Journal of Physics: Conference Series (2021).
- [9] J. Lu, "The Research on PID Control of Three-Tank Water System Based on BPNN, "Hefei University of Technology (2009).
- [10] S. Yu, H. Chen, P. Zhang,R. Sun, and M. Shi, "Moving Horizon  $H_{\infty}$  Control of a Three-Tank System and Its Experiment Study, "Journal of Northeastern University (Natural Science) (2007), 28,pp. 82-91.