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The application of the absorbing Markov chain in analysing the movement of the students on the faculty of science-university of Tobruk

Enas H Abduallah [1], Mahmoud Saed Asbeetah[2]

Mathematics department, Tobruk University, Tobruk, Libya.
enas.hamed@tu.edu.ly, mahmoud.asbeetah@tu.edu.ly

Abstract

The research presented the application of the absorbing Markov chain as a method of stochastic process in analysis the movement of students of the faculty of science at the University of Tobruk between (2012/2013-2023/2024). The results showed that the annual average number of graduations is 94 and the annual graduation rate is 72% according to the enrolment of enrolment students this year. In addition, the average time of 67% freshman, 82% sophomore, 91% junior and 96%senior to stay at faculty science plus the current year until obtaining a bachelor's degree (graduation) is (three, two, one and a half and year) respectively. Furthermore, the average number of students who expect to obtain a bachelor's degree in the next four years is 577.

Keywords: stochastic process, absorbing Markov chain, transition matrix.

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تطبيق سلاسل ماركوف الامتصاصية لتحليل حركة طلاب كلية العلوم بجامعة طبرق

إناس حمد عبدالله[1]، محمود سعيد اسبيته[2]
قسم الرياضيات / جامعة طبرق ليبيا
mahmoud.asbeetah@tu.edu.ly, enas.hamed@tu.edu.ly

الملخص
تناول هذا البحث تطبيق لسلاسل ماركوف الماصة كأحد طرق العمليات العشوائية لتحليل حركة طلبة كلية العلوم بجامعة طبرق للمدة الزمنية ما بين 2013/2012 إلى 2024/2023. أظهرت النتائج ان متوسط عدد الخريجين السنوي هو 94 بينما معدل التخرج السنوي هو 72% طبقاً لأعداد المسجلين لتلك السنوات. بالإضافة إلى ذلك الزمن المستغرق لحوالي 67% من طلبة المرحلة الأولى، ولحوالي 82% من طلاب المرحلة الثانية، كذلك ل 91% من طلاب المرحلة الثالثة وكذلك لحوالي 96% من طلاب المرحلة الرابعة حتى تخرجهم وحصولهم على درجة البكالوريوس بالإضافة إلى السنة الدراسية الحالية هو (ثلاث سنوات، سنتان، سنه ونصف، سنه واحده) على التوالي. كذلك العدد المتوقع لأعداد الطلبة المتخرجين لأربع سنوات قادمة هو 577 طالب تقريباً.

الكلمات المفتاحية: العمليات العشوائية، سلاسل ماركوف الماصة، المصفوفة الانتقالية.

Introduction

By comparing the high education in Libya to developing countries, there was almost no higher education system at all in the last few decades ago, although Libya has education free of charge for all stages. As a result, about 70% of men and 35% of women were literate in the early 1980's. This changed to literacy for men above 90%, and more than 70% for women in 2004 [1].

The education system in Libya must achieve a high standard. Creative solutions must be applied to challenges and problems, like new technologies, updated syllabus and quality assurance in education. Further, a true plan with clear targets must be undertaken, and a post-graduation system should be established [2].

The Absorbing Markov chain is the most important mathematical and statistical method that could be used to analyse and plan for universities. In addition, the Markov chain model is used for the

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expected duration of study, and moreover, making the decision to take advantage of future planning [3].

The University of Tobruk was chosen to apply the Markov chain. It specially targeted the faculty of science between (2012/2013-2023/2024) because of the importance of the faculty of science in the job market. In addition, to resolving the problems of unemployment, through predicting high education outcomes and promoting decision-making for planning high education.

Research objectives

1- Estimate the lifetime of a student for graduation,
2- Estimate the probability of student graduation,
3- Estimate the probability of a student dropout,
4- Predict the number of graduations and the number of students who they are expected to drop out.

Literature review

1- Alenka and Mirjana (2017): The paper aimed to develop a stochastic model for estimation and continuous monitoring of various quality and effectiveness indicators of a given higher education study program. The model was applied to study the performance of students' enrolment and their academic achievement in a Slovenian higher education institution between 2008 and 2017. The study conclude that the model enables estimation and continuous monitoring of different quality and effectiveness indicators of a given study program, furthermore the probability of graduation and withdrawal was obtained as well as the prediction of graduation of next three years [4].
2- Egbo, Bartholomew and Okeke (2018): The paper used Markov Chain to develop an enrolment projection model for Apostolic Faith Secondary School, Akwa Ibom State, Nigeria (between 2008/2009-2013/2014 academic sessions). The research concluded that the model is useful for the school's future planning [5].
3- Khairun and Husna (2021): The main reason of the study was to analysis the study plan of students' assessment and their academic performance in School of Mathematical Sciences, University Sains Malaysia. The aimed population of the study was all undergraduate enrolment from 2016/2017 until 2018/2019 sessions. Markov chain was used to describe the stochastic pattern of enrolments and assessment of students. In addition, the model was designed to study the absorption,

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retention and repetitive rates of the students by the academic programs according to the gender of the students. The research included that the Markov chain model is effective for illustrating he probabilistic behavior of students' enrolment data [6].

Theoretical aspect

Markov chains are a type of stochastic process that they are applied in areas such as education, marketing, health services, finance, accounting and production.

1- Stochastic process

Let $X_t$be the value of the system characteristic at time t where t a system at discrete points in time (labeled 0, 1, 2, ….) discrete time. In most situations, $X_t$is not known with certainty before time t and may be viewed as a random variable. The definition of discrete-time stochastic process is simply a description of the relation between the random variables $X_0, X_1, X_2,$ .... clearly a stochastic process in which the state of system can be noticed at any time, not just at discrete current in time.

2- Markov Chain

A discrete time stochastic process is called a Markov chain if, for $t = 0,1,2, \ldots$ and all states,

$$P(X_{t+1} = i_{t+1} | X_t = i_t, X_{t-1} = i_{t-1}, \ldots \ldots, X_1 = i_1, X_0 = i_0) = P(X_{t+1} = i_{t+1} | X_t = i_t \quad (1)$$

It means that the probability distribution of the state at time $t + 1$ based on the state at time $t(i_t)$ and does not based on the state the chain passed through on the way to $i_t$at time t.
To make further assumption of Markov chains for all states i and $j$and all t, $P(X_{t+1} = j | X_t = i$is independent of t. This assumption granted us to write:

$$P(X_{i+1} = j | X_t = i) = P_{ij} \quad (2)$$

Where: P_{ij} is the probability that given the system in state i at time t, it will be in state j at time $t + 1$.
If the system moved from state i during one period to state j during the next period, the P_{ij}'s are often defined as the transition probabilities for the Markov chain.
From equation (2), if the current state does not change (or remain stationary) over time, the equation (2) is often called the Stationarity

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Assumption. Furthermore, any Markov chain that satisfies the equation is called stationary Markov chain.

The probability of the chain is in state i at time 0; in other words, $P(X_0 = i) = q_i$. Where: $q = [q_1 q_2 \dots q_s]$ the initial probability distribution for the Markov chain. The transition probabilities are displayed as an $s \times s$ transition probability matrix P which is written as:

$$P = \begin{bmatrix} p_{11} & p_{12} \cdots & p_{1s} \\ p_{21} & p_{22} \cdots & p_{2s} \\ \vdots & \vdots & \vdots \\ p_{s1} & p_{s2} \cdots & p_{ss} \end{bmatrix}$$

Given that the state at time t is i, the process must somewhere at time $t + 1$ this means that for each i,

$$\sum_{j=1}^{j=s} P(X_{t+1} = j | P(X_t = i)) = 1$$

$$\sum_{j=1}^{j=s} P_{ij} = 1$$

Each entry in the P matrix must be positive. Hence, all entries in the transition probability matrix are positive, and the inputs in each row must sum to 1[7].

All the element of matrix P_{ij} of Markov chains represents the transition from i to $j$after times period its amount nP_{ij}^n

$$P_{ij}^n = P[X_{m+n} = j | X_m = i] \quad (3)$$

It is possible to write transition probabilities after n steps by

$$P^{(n)} = \begin{bmatrix} p_{11}^{(n)} & p_{12}^{(n)} \cdots & p_{1s}^{(n)} \\ p_{21}^{(n)} & p_{22}^{(n)} \cdots & p_{2s}^{(n)} \\ \vdots & \vdots & \vdots \\ p_{s1}^{(n)} & p_{s2}^{(n)} & p_{ss}^{n} \end{bmatrix}$$

Where:

- If $n = 1$ the probability of transition from i to j becomes after one step like as P_{ij}.
- If $n = 0$ then [3].

$$P_{ij}^n = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

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3- Absorbing chains

A Markov chain in the many applications involves chains in which some of the states are absorbing and the rest are transient states. The chain called absorbing if it begins in a transient state, then eventually it is sure to leave the transient sate and end up in one of the absorbing states.

The transition matrix for the absorbing chain may be written as followers:

$$P = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix} \quad (4)$$

Where:

P : is correspond to the state $t_1, t_2, \dots, t_{s-m}, a_1, a_2, \dots, a_m$,

I : is an $m \times m$ identity matrix reflecting the fact that we can never leave an absorbing state,

Q : is an $(s-m) \times (s-m)$ matrix that represents transition between transient states,

R: is an $(s-m) \times m$ matrix represents transitions from transient states to absorbing state,

0: is an $m \times (s-m)$ matrix consisting of zeros.

This reflects the fact that it is impossible to go from an absorbing state to transient state [7].

It can be concluded that:

$$P^2 = \begin{bmatrix} Q^2 & R(I+Q) \\ 0 & I \end{bmatrix}$$

$$P^3 = \begin{bmatrix} Q^3 & R(I+Q+Q^2) \\ 0 & I \end{bmatrix}$$

After n steps:

$$P^n = \begin{bmatrix} Q^n & R(I+Q+Q^2 \dots Q^{n-1}) \\ 0 & I \end{bmatrix}$$

Then:

$$\lim_{n\to\infty} P^n = \begin{bmatrix} Q^n & R(I-Q)^{-1} \\ 0 & I \end{bmatrix}$$

Where:

- $Q^n \to 0, n = \infty$
- $\sum_{i=0}^{\infty} Q^i = I + Q + Q^2 + \dots \dots \dots = (I-Q)^{-1}$ (5)

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$(I - Q)^{-1}$ is the Markov chain's fundamental matrix and it is symbolized with N. In addition, the probability matrix of transited from transient state to absorbing state is symbolized with B
Where:

$$B = (I - Q)^{-1}.R = N.R \quad (6)$$

The average absorbing time from the transited state is symbolized with M [3].
Where:

$$M = (I - Q)^{-1}.I = N.I \quad (7)$$

The matrix F shows the predicted number of students for absorbing state after specific time M depending on the number of students at specific time i (W_i) [8].
Where:

$$F = W.B \quad (8)$$

Applied aspect
The main resource for the data is the results record of the students in each year from (2012/2013–2023/2024) and the archive of the admissions and study unit at the faculty of sciences.
Based on the data, there are seven situations for the Markov matrix that are divided into two parts: five situations are transient states and two are absorbing states, as follows:

L_1	The state of the freshman	transient states
L_2	The state of the sophomore	
L_3	The state of the junior	
L_4	The state of the senior	
L_I	The state of transfer in & out	
L_{II}	The state of Expulsion	absorbing states
L_{III}	The state of graduation	

Table (1) illustrates the numbers of the students who enrolled in the four levels and the number of graduates, in addition to the number of students who stay at the same level, transferred in and out and expelled. Furthermore, the faculty of science follows the regulations number (501) for the year 2010 to enroll, pass and expel students.

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Table1 shows the number of students and the graduates between 2012/2013 – 20123/2014

Table 1: The number of students enrolled in and the graduates

Academic year	freshman	Sophomore	junior	Senior	Graduates
2012/2013	353	-	-	-	-
2013/2014	320	244	-	-	-
2014/2015	422	215	191	-	-
2015/2016	364	203	163	225	144
2016/2017	136	233	179	181	134
2017/2018	167	96	178	169	89
2018/2019	160	64	83	202	153
2019/2020	137	88	65	107	88
2020/2021	115	83	79	67	59
2021/2022	-	76	67	74	62
2022/2023	-	-	57	75	58
2023/2024	-	-	-	67	63
∑	2174	1302	1062	1167	850

The average of the senior = $\frac{1167}{9} \cong 130$

The average of the graduates =$\frac{850}{9} \cong 94$

The annual graduation rate =$\frac{94}{130} \times 100 = 72.30\%$

Table 2 illustrates the number of students who they are stayed in at the same level

Table 2: The number of students (staying in)

Academic year	Freshman	Sophomore	Junior	senior
2012/2013	80	-	-	-
2013/2014	66	45	-	-
2014/2015	52	31	15	-
2015/2016	45	36	34	65
2016/2017	24	39	34	46
2017/2018	29	12	32	54
2018/2019	59	11	17	44
2019/2020	26	6	4	10
2020/2021	13	21	10	5
2021/2022	-	11	6	12
2022/2023	-	-	12	16
2023/2024	-	-	-	4
∑	394	212	164	256

- The probability of staying in the first level is P_{s1}

$$P_{s1} = \frac{394}{2174} = 0.18$$

- The probability of staying in the second level is P_{s2}

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$$P_{s2} = \frac{212}{1320} = 0.16$$

- The probability of staying in the third level is P_{s3}

$$P_{s3} = \frac{164}{1062} = 0.15$$

- The probability of staying in the third level is P_{s4}

$$P_{s4} = \frac{256}{1167} = 0.21$$

Table 3 displays the number of students who transfer out during the study period

Table 3: The number of student (Transfer out)

Academic year	Freshman	Sophomore	junior	senior
2012/2013	61	-	-	-
2013/2014	87	19	-	-
2014/2015	132	35	35	-
2015/2016	138	26	0	0
2016/2017	48	42	18	0
2017/2018	73	25	0	0
2018/2019	20	2	0	0
2019/2020	27	4	3	0
2020/2021	39	7	0	0
2021/2022	-	5	0	0
2022/2023	-	-	0	0
2023/2024	-	-	-	0
∑	625	165	56	0

- The probability of transfer out from the first level is P_{1I}

$$P_{1I} = \frac{625}{2174} = 0.28$$

- The probability of transfer out from the second level is P_{2I}

$$P_{2I} = \frac{165}{1302} = 0.12$$

- The probability of transfer out from the third level is P_{3I}

$$P_{3I} = \frac{56}{1062} = 0.05$$

- The probability of transfer out from the fourth level is P_{4I}

$$P_{4I} = \frac{0}{1167} = 0$$

Table 4 shows the number of students transfer in through study time

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Table 4: The number of students (transfer in)

Academic year	Freshman	Sophomore	Junior	senior
2012/2013	7	-	-	-
2013/2014	6	2	-	-
2014/2015	2	0	0	-
2015/2016	3	0	0	0
2016/2017	1	0	0	0
2017/2018	2	0	0	0
2018/2019	1	2	0	0
2019/2020	1	3	0	0
2020/2021	2	0	0	0
2021/2022	-	1	0	0
2022/2023	-	-	0	0
2023/2024	-	-	-	0
$\sum$	25	8	0	0

- The probability of transfer in from the first level is P_{I1}

$$P_{I1} = \frac{25}{33} = 0.75$$

- The probability of transfer in from the second level is P_{I2}

$$P_{I2} = \frac{8}{33} = 0.24$$

- The probability of transfer in from the third level is P_{I3}

$$P_{I3} = 0$$

- The probability of transfer in from the fourth level is P_{I4}

$$P_{I4} = 0$$

Table 5 explains the number of student expulsions through the study time

Table 5: The number of students (expulsion)

Academic year	Freshman	Sophomore	Junior	senior
2012/2013	22	-	-	-
2013/2014	23	12	-	-
2014/2015	86	19	1	-
2015/2016	54	7	4	7
2016/2017	13	13	9	12
2017/2018	17	6	6	9
2018/2019	7	0	7	3
2019/2020	9	7	4	9
2020/2021	12	3	3	3
2021/2022	-	3	0	0
2022/2023	-	-	0	1
2023/2024	-	-	-	0
$\sum$	243	70	34	44

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- The probability of expulsion from the first levelP_{1II}

$$P_{1II} = \frac{243}{2174} = 0.11$$

- The probability of expulsion from the second levelP_{2II}

$$P_{2II} = \frac{70}{1320} = 0.05$$

- The probability of expulsion from the third levelP_{3II}

$$P_{3II} = \frac{34}{1062} = 0.03$$

- The probability of expulsion from the fourth levelP_{4II}

$$P_{4II} = \frac{44}{1167} = 0.03$$

Depending on the previous probabilities, the equations of transfer between academic levels are:

- The probability of transfer from freshman to sophomore is P_{12}:

$$P_{12} = 1 - [P_{s1} + P_{1II} + P_{1I}]$$
$$= 1 - [0.18 + 0.11 + 0.28] = 0.43$$

- The probability of transfer from sophomore to junior is P_{23}:

$$P_{23} = 1 - [P_{s2} + P_{2II} + P_{2I}]$$
$$= 1 - [0.16 + 0.05 + 0.12] = 0.67$$

- The probability of transfer from junior to senior is P_{34}:

$$P_{34} = 1 - [P_{S3} + P_{3II} + P_{3I}]$$
$$= 1 - [0.15 + 0.03 + 0.05] = 0.77$$

- The probability of transfer from senior to graduated is P_{4III}:

$$P_{4III} = 1 - [P_{s4} + P_{4II} + P_{4I}]$$
$$= 1 - [0.21 + 0.03 + 0] = 0.76$$

From preceding probabilities, we can format the transition matrix for the absorbing chain P as equation 4.

$$P = \begin{array}{c} \\ L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_I \\ \\ L_{II} \\ L_{III} \end{array} \begin{array}{c} \quad L_1 \; L_2 \qquad L_3 \qquad L_4 L_I L_{II} L_{III} \\ \begin{bmatrix} 0.18 & 0.43 & 0 & 0 & 0.28 & \vdots & 0.11 & 0 \\ 0 & 0.16 & 0.67 & 0 & 0.12 & \vdots & 0.05 & 0 \\ 0 & 0 & 0.15 & 0.77 & 0.05 & \vdots & 0.03 & 0 \\ 0 & 0 & 0 & 0.21 & 0 & \vdots & 0.03 & 0.76 \\ 0.75 & 0.24 & 0 & 0 & 0 & \vdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & \vdots & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \vdots & 0 & 1 \end{bmatrix} \end{array}$$

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Markov chain's fundamental matrix N following equation5.

$$N = (I - Q)^{-1} = \begin{bmatrix} 1.866 & 1.157 & 0.912 & 0.889 & 0.707 \\ 0.278 & 1.419 & 1.119 & 1.090 & 0.304 \\ 0.086 & 0.071 & 1.232 & 1.201 & 0.094 \\ 0 & 0 & 0 & 1.265 & 0 \\ 1.466 & 1.208 & 0.952 & 0.928 & 1.603 \end{bmatrix}$$

From equation 7 the average absorbing time from the transited state M is calculated as follows:

$$M = \begin{bmatrix} 5.531 \\ 4.212 \\ 2.685 \\ 1.265 \\ 6.159 \end{bmatrix}$$

The result of matrix M shows the average expected cumulative stay time for the students in each level and the students who transferred into the faculty until their graduation from the faculty of science as follows:

Table 6: Stay time for the students in each level

Academic level	Average cumulative stay time	Approximate average cumulative stay time
Freshman	5.531	3 years
Sophomore	4.212	2 years
Junior	2.685	1.5 year
Senior	1.265	1 year
Transfer in the faculty	6.159	3 years

The probability matrix of transited from unabsorbing state to absorbing state is B computed as equation 6.

$$B = \begin{bmatrix} 0.317 & 0.675 \\ 0.167 & 0.829 \\ 0.086 & 0.913 \\ 0.038 & 0.962 \\ 0.278 & 0.705 \end{bmatrix}$$

To predict the number of students who expected to graduate or expulsion for next four years after the study time:
Table 6 shows the number of students in the academic levels

Table 7: The number of students in 2023/2024

Freshman	Sophomore	Junior	Senior
396	158	126	67

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Calculating F by producing the vector W with B following equation 8.

$$W = [396 \quad 158 \quad 126 \quad 67]$$

$$F = [396 \quad 158 \quad 126 \quad 67]\begin{bmatrix} 0.317 & 0.675 \\ 0.167 & 0.829 \\ 0.086 & 0.913 \\ 0.038 & 0.962 \end{bmatrix}$$
$$= [165.300 \quad 577.774]$$

$$\cong [165 \quad 578]$$

The average number of students will graduate in next four years between (2024/2025-2027/2028) is 578, also the average number of students will expulsion is 165 at the exact time, this is demonstrated in table (8):

Table 8: The average number of students will graduate or expulsion in next four years

Academic year	Graduation average	Expulsion average	Last situation
2024-2025	64	126	Senior
2025-2026	115	26	Junior
2026-2027	131	11	Sophomore
2027-2028	267	3	Freshman
$\sum$	577	166	

Conclusion

The most important results that Markov Chain approved are

- The annual average number of graduations is 94 and the annual graduation rate is 72% according to enrolment students this year.
- The average time of 67% freshman's students stay at the faculty of science plus the current year until obtaining a bachelor's degree (graduation) is three years with a probability of 0.31 of getting expelled.
- The average time of 82% of sophomore's stay at the faculty of science plus the current year until obtaining a bachelor's degree (graduation) is two years with a probability of 0.16 of getting expelled.
- The average time of 91% of juniors stay at the faculty of science plus the current year until obtaining a bachelor's degree

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(graduation) is a year and a half with a probability of 0.08 of getting expelled.
- The average time of 96% of seniors stay at the faculty of science plus the current year until obtaining a bachelor's degree (graduation) is a year with a probability of 0.03 of getting expelled.
- The average time of students who transfer to the faculty of science from the current year until obtaining a bachelor's degree (graduation) is three years.
- The average number of students who are expected to obtain a bachelor's degree in the next four years (2024/2025-2027/2028) is 577.
- The average number of students who are expected to be expulsion in the next four years is 166.

Recommendation

- We recommend the researcher to pay attention to prediction using the Markov chain because it is not affected by affecting factors.
- Appley Markov chains with other faculty at the university of Tobruk as a mechanism to expect the number of graduates.
- Appley Markov chain as a mechanism to predict the number of graduates in the rest of the universities of Libya.
- We recommend decision-makers to map out a strategy for university graduates according to Markov chain results to avoid unemployment.

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